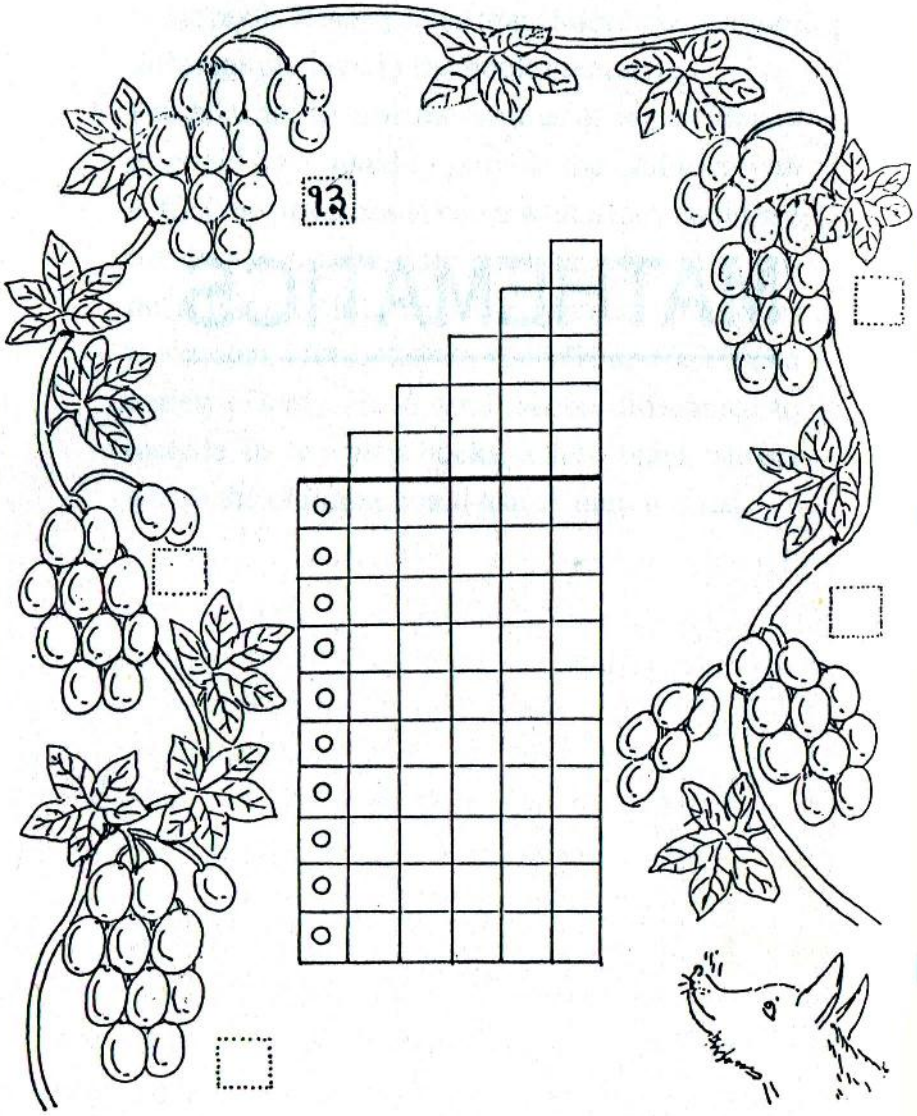
MATHEMATICS



A page from KHUSHI-KHUSHI . . .

PRE-PROGRAMME TESTING

Eklavya's earlier experience in developing the middle school science curriculum had revealed the low level of mathematical skills of children coming from primary schools. Prashika decided to conduct a series of tests in maths from Class I through V to assess specific learning problems.

The tests in mathematics focused on numbers and the four basic operations as well as on spatial ideas like translating, rotating, estimating, etc. Prashika not only attempted to see whether children could do 'sums' but also whether they understood what they were doing.

The only skill that most children seemed to have mastered by Class V was simple addition of numbers upto 10. It is because of this that they could get the correct answer even in 3-digit addition which did not require carrying over. It was only when carry-over additions (where the concept of place value comes into use) were attempted by these children that it became evident that the concept of numbers beyond 10 is not clear to them. The most common error found was of the kind where each column was added independently.

Addition seemed to be so strong in the minds of children that even when asked to do division or

5	7
7	5
12	12

multiplication, they simply added. Fewer children attempted operations presented horizontally, for example, 65 – 23 = 42. Story problems and contextual problems seemed the most difficult – very few children managed to apply the right algorithm.

Skills of addition and subtraction seemed to improve from Class III onwards but understanding of number order and place value showed no change.

As for fractions and decimals, which are taught in Classes III and IV, only about 10% of the children tested could even mechanically attempt the problems. The problems on spatial skills too were rarely attempted.

To try and understand the reason for this deplorable state of affairs, the curriculum and textbooks of primary schools were reviewed and classroom teachers were observed.

THE STATE CURRICULUM

The state curriculum for Classes I to V is extremely dense. It does not allow children any space to come back to what they have already learnt and may have forgotten. For example, in Class I children are sup-

posed to have learnt numbers from 1 to 100, all the four operations on numbers 1 to 100 and multiplication tables till 10.

The state curriculum is marked by lack of reinforcement. In Class II there are about 10 pages revising the Class I curriculum. Thereafter, the chapters move on to larger numbers (up to 1000), and newer ideas like weight and time units etc., never once returning to earlier principles and concepts, which, it was assumed, had been learnt by all the children. Intensive exploratory discussions with children showed that they were not clear about many concepts taught in the earlier classes and had never had the opportunity to re-examine these earlier concepts. It became increasingly clear that the Prashika curriculum will have space for inbuilt horizontal elaboration where children could return to the same concepts again and again.

Moreover, the approach in the textbooks is extremely mechanical. A solved example of each type is given and then a number of exercises follow. The contexts for the problem are usually unfamiliar and uninteresting. Finally, most teachers are disinterested in the subject and teach it in a very mechanical manner. Given all this it is not surprising that children develop a fright for mathematics at an early age.

Some Basic Principles

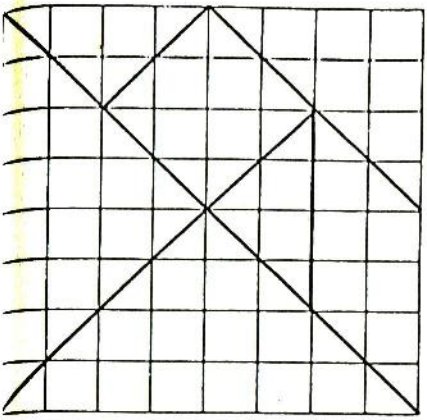
Prashika acknowledges a fundamental link at the cognitive level between language and mathematics learning. In Classes I and II especially there are many common activities which have been found to serve as building blocks for both language and mathematics learning.

The programme does not encourage meaningless memorization, especially of abstract rules and algorithms. It is believed that, given a certain experience base, children *can* understand the rules behind many aspects of mathematics.

For this it is necessary to emphasize, especially in the earlier phases, activities that involve interaction with concrete objects. In later phases, articulation of the method of doing a sum or converting problems with numbers to verbal problems could be emphasized to encourage understanding. Thus, reflection on experience rather than drill is considered the vehicle to mathematical understanding.

As part of the overall cognitive development of the child, mathematics in Prashika lays equal emphasis on the development of spatial and numerical skills. Finally, mathematics is seen as more than a subject. It is a way of looking at the world around, and understanding it in quantitative terms.

टेनग्राम

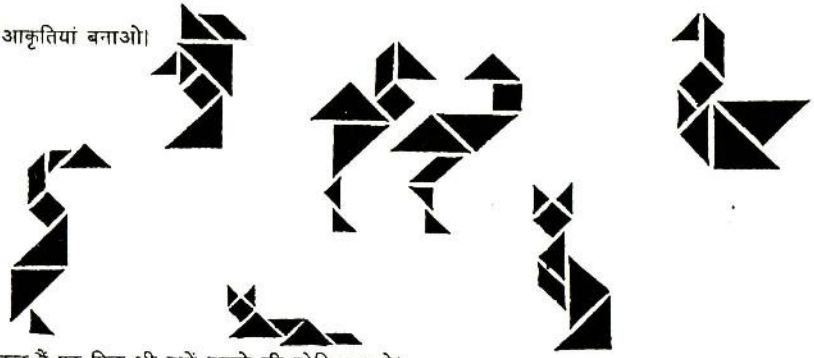


यहां चौखानों की एक जाली दी गई है। पृष्ठ पर दिए गए चौखाने कागज पर इतना ही बड़ा एक चौकोर काट कर उसे एक पुट्ठे या गत्ते पर चिपका लो।

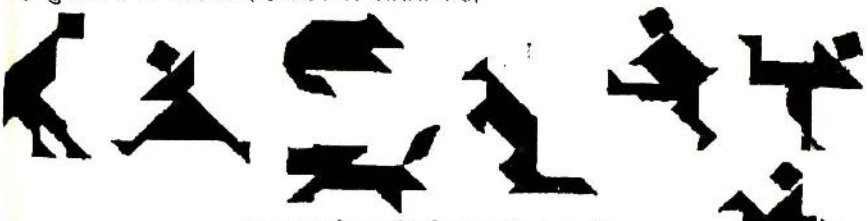
अब जाली में गहरी लाइनों से बनी आकृतियां अपने चौकोर में उतार लो। इन लाइनों पर काट कर सात टुकड़े बना लो। ये टुकड़े ऐसे दिखेंगे।

टेनग्राम से तुम कई तरह की आकृतियां बना सकते हो। 2 या 3 टुकड़ों से एक तिकोन बनाने की कोशिश करो। और कितनी तरह से तिकोन बनता है? बनाकर देखो। चतुर्भुज किस-किस तरह से बना सकते हो?

अब तुम ये आकृतियां बनाओ।



थोड़ी मुश्किल हैं पर फिर भी इन्हें बनाने की कोशिश करो।



● अब अपने मन से और आकृतियां बनाओ।

THE PRASHIKA CURRICULUM

A major way in which the Prashika curriculum differs from the state curriculum is that the levels of expectations regarding numbers are pitched at a drastically lower and more realistically achievable level in earlier classes. For instance, it is not expected that children in Class I will be able to go beyond 20. At the same time the emphasis is not merely on being able to repeat the number sequence in abstraction but actually being able to count a set of objects, which is never emphasized in the state curriculum.

At the same time, mathematics right from Class I is seen as more than number work. It broadly includes

1. number,
2. space and shape,
3. other ways of handling data – maps, pictographs etc., and
4. measurement.

Keeping in mind the age and background of the children who come to Prashika schools, a long time is devoted to precounting activities like sorting, classifying, one-to-one correspondence and sequencing. Most of these are done using concrete materials from the environment.

Also, and this is particularly true in the case of numbers, there is a mathematics to be learnt in the

context of everyday life and the physical world. In addition, there is also a mathematics to be learnt in abstraction, for example, patterns and relationships between numbers or within a given system.

While Prashika accepts that the first precedes the second, and that the second derives from the first, it also believes that abstraction need not be postponed till children are much older (i.e. till middle school).

THE SEQUENCE OF LEARNING

Prashika recognizes various levels of mathematical capabilities. These include

1. 'initial' or 'intuitive' mathematics; intuitive because it precedes, and can exist independently of, any formalizations, either in terms of symbolic notations or defined operations. It is linked to an operative relationship with the world.
2. An initial stage where concrete objects and visuals are necessary to explore mathematical concepts and to conceptualize.
3. A pre-logical stage in the development of mathematical cognition and the need for readiness tasks in the curriculum.

CONTEXTUAL ELABORATION

Prashika makes a concerted effort not to rush children

ACCORDING TO PRASHIKA . . .

As children who study in Prashika schools have never been to pre-school institutions, the first six months in Class I are spent in activities – without using the book at all. These include sorting, matching, ordering. . . . The book itself does not attempt to teach children 'numbers'. Number sequences are learnt orally by the child from teachers, from peers, and from the community at large. Instead, the book provides opportunities to use the number scheme repeatedly both to assess cardinality and ordinality. Often the number of objects/items to be counted exceed 10. There is a belief that initially children should deal with numbers only below or upto 10 and that their exposure to larger numbers should be graded on increasing powers of ten. However, these limits are found to have no psychological significance. The counting scheme can be exercised independently of these limits, and to some extent operations such as addition, subtraction, multiplication and even division can be successfully performed without taking any special recourse to algorithms. However Prashika does not expect children to decode numbers written above 9 or to extend these operations to larger numbers. This coincides with a psychological finding that numbers upto 6 or 7 are intuitively understood by children and that they also possess intuitive ways of dealing with such small 'visual' numbers.

through the abstract concepts of mathematics. For example, the concept of place value is explored in a variety of contexts. Some specific examples from the workbook are:

1. Class I. The base ten nature of number naming/ notation system is not even hinted at, except on the last page of *Khushi-Khushi I*, where a histogram-like picture is used to suggest that 11, 12, ... 15 may be thought of as 10 + 1, 10 + 2 etc. *Ikai, dahai* are not used at all.
2. Class II. Place value is explored in a concrete, contextualized set of activities. These activities are intended to have children redefine numbers around 10. The words for units and tens are rarely used. Children are not expected to use them in any abstract sense.
3. Class III. Here children do rewrite numbers with respect to place value in the abstract sense. The concept of 'borrowing' and 'lending', i.e. addition and subtraction algorithms are explored through a game based on exchanging 10 beads for a card.
4. Class IV. There are Units-Tens-Hundreds (UTH) exercises in contextualized and decontextualized forms. The multiplication algorithm is explored through the idea of decades. The number charts

provide enormous elaboration of base 10. Card-bead games are still used.

5. Class V. There is an attempt to explain the division algorithm on the basis of UTH. There are exercises of writing numbers in 'expanded' form etc. Decimals and how they can be regarded as an extension of the place value system to the right of the units place is also referred to.
6. Similarly the development of shapework, mapwork, and numbers as an abstraction are found to be different from the dominant ways of dealing with these areas in the usual textbooks.

METHODS

Prashika believes in allowing children to experiment and to explore ideas. Children should think not only to get answers, but also to understand processes and underlying methods. Some of the basic methodological assumptions of Prashika are as follows:

1. Only if the teachers begin to enjoy mathematics will they be able to further communicate some of this enjoyment.
2. Children should get ample opportunities for concrete activities based on mathematical concepts. At the same time, Prashika recognizes the limitation of using concrete materials for abstract con-

cepts and suggests interesting activities to explore them at an early stage.

3. Children should be given the opportunity to discuss how they are doing something in mathematics since articulation is an integral part of understanding.
4. There should be plenty of coming back to first principles. Concepts should be reintroduced in different contexts at different steps so that children who could not catch on in the first instance have an opportunity later on.
5. Children should get the opportunity not only to solve problems given by the teacher, but also to hypothesize and to make their own problems.
6. Children make mistakes for a variety of reasons including the systems in transition they have made for themselves. It is therefore important to try to understand *why* they make mistakes. Talking about errors is far more important than trying to rectify errors at one stroke. Errors that are overcome through discussion and attempts at conceptual clarity are likely to disappear. Teachers must not so much emphasize the *right* procedure, but allow for expression of and exploration of other procedures.

MATERIALS

As already pointed out, there is no book for the first six months of Class I. The focus is on activities with concrete materials, picture cards and outdoor activities. The activities and units in the workbook are meant as examples for the teacher so that she can create more of her own activities as and when necessary.

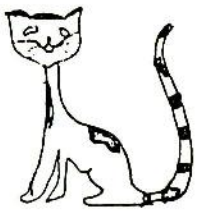
The Class I book provides opportunities for mathematical activities like counting, addition and subtraction, familiarity with shapes, etc. on nearly every page. Though there are a few pages specifically meant for mathematical activities, we encourage the teachers to create interesting contexts through stories and discussions for teaching mathematical operations.

In the later classes, stories often have questions that require the application of mathematical skills along with questions on comprehension.

Prashika tries to focus on relationships between numbers and operations, discovering patterns in numbers as well as spatial relationships. In such activities Prashika asks children to test their hypothesis about such relationships and also to make up more similar questions themselves.

Fairly simple problems of addition and subtraction

इस तरह से आकार को नियमित रूप से बराबर अनुपात में बदलने को स्केल बदलना कहते हैं। स्केल बदलने यानी पैमाना बदलने में हम तुलना का मानक बदल देते हैं। जैसे, हमने ऊपर दिए नक्शों में दिखाया है कि एक कदम का मान एक तीली, आधी तीली या एक बीज – कुछ भी हो सकता है।



उदाहरण के लिए, बैठी हुई बिल्ली की ऊंचाई 50 से.मी. (यानी डेढ़ फुट से अधिक) होती है। इस कागज़ पर तो इतनी बड़ी बिल्ली का चित्र बन नहीं सकता। चलो हम 50 से.मी. को 1 से.मी. के बराबर मान कर चित्र बनाए। अर्थात इस चित्र के लिए हमारा स्केल है – 50 से.मी.= 1 से.मी.

यदि हमारा स्केल 50 से.मी. = 2 से.मी. हो तो बिल्ली और भी छोटी बनेगी।

यह मध्यप्रदेश का नक्शा है। इस नक्शे की चौड़ाई 6 से.मी. है।

अब 3 से.मी. चौड़ा मध्यप्रदेश का नक्शा बनाओ। 3 से.मी. चौड़े नक्शे में 'क ख' लम्बाई कितनी होगी? और भोपाल से इंदौर की दूरी कितनी?

SEQUENCING IN KHUSHI-KHUSHI . . .

The layout and nature of materials in the Khushi-Khushi books also indicate certain assumptions about sequencing:

1. There is a developmental agenda for cognitive structures – a certain time-table according to which they develop. Therefore not only do certain mathematical concepts need to be postponed till later, but they also need to be elaborated in a particular manner.
2. There is a logical (hierarchical) sequencing which is dictated by the discipline itself, for example, addition precedes multiplication, etc. However, this sequencing is tempered by some considerations about the nature of learning.
3. Learning is not linear – there should be looping, i.e. opportunities to return to simpler earlier concepts and operations, and also opportunities to attempt things judged to be at a higher level.
4. The rate at which new ideas, concepts and operations are introduced is also not linear: it is much slower in Classes I and II and gradually proceeds at an increasing pace later.

are continued even in Classes III and IV but they are different from those done in Classes I and II. The objective of this is both to give an opportunity to

children to re-examine earlier concepts as well as for them to discover new relationships between numbers and their operations. Children are encouraged to make word problems based on mathematical operations and their application.

Practice and horizontal elaboration are very important in mathematics. However, the books themselves have only a limited number of exercises for children to do. Prashika has tried to use other ways of generating more exercises for children by:

1. using games such as housie, snakes and ladders etc. which are open ended and can generate many problems. With slight modifications such games can also be extended to involve higher operations and concepts.
2. asking children to make up problems and quiz each other.
3. training teachers to make up more problems for children.
4. using number charts etc. where there are many possible patterns and relationships that can occupy children.
5. providing children with a challenge through statements like '*Mera dava hai ki . . .*' (I claim etc.) so that now they have to investigate.

THE TEACHER

In most of the primary schools teaching was found to be as mechanical and uninteresting as the curriculum and the textbooks. It is not the fault of the teacher. The amount (s)he has to teach in the limited period of a school year, with the children's attendance varying with the seasons, prohibits any imaginative teaching.

The primary school teacher has usually passed high school (often without mathematics as a subject). Sometimes (s)he has studied only upto Class VIII. Most teachers are allergic to mathematics and have never enjoyed it themselves. Nor have they been oriented properly to teach mathematics to very young children. They have never been sensitized towards children's problem in learning mathematics.

Prashika regards teacher involvement in the curriculum development process as an integral part of its programme. It constantly interacts with teachers for curriculum development and review, pedagogy, evaluation problems, etc.

During teacher-orientation sessions, flexible use of the curriculum as well as the workbooks is emphasized. Teachers suggest and exchange among themselves a number of different activities, stories etc. which can serve to fulfil a certain cognitive aim.

Another aspect emphasized during the sessions of

teacher orientation is the importance of children's contribution of their own knowledge in the learning process. In these interactions, the teacher begins to appreciate that the children are not empty vessels and that making mistakes is an essential part of the learning process. In the orientation programmes the emphasis is not on providing a readymade correct answer but to explore different paths to arrive at possible solutions. There are a number of activities during the orientation, where no one actually provides 'right' answers but only guides a discussion and different teachers contribute different points of information. It is only after a number of such orientation sessions and after the teachers have gone back to school and tried out such activities with children and seen what they can contribute, that about a third of the teachers begin to re-examine their views.

THE OUTCOME

The atmosphere in the Prashika classroom now is a pleasant departure from the usual disciplined class. Children of Classes IV and V can now rattle off a number of extremely imaginative problems involving a particular operation. They can tell you how they solve a problem and why they are doing it that way. Though most Class V children cannot add and sub-

tract fractions, they can probably tell you which fraction is larger in a number of different ways.

Their spatial skills too have improved considerably. They can make and read maps, estimate area and volume, and even point out minute differences between pictures.