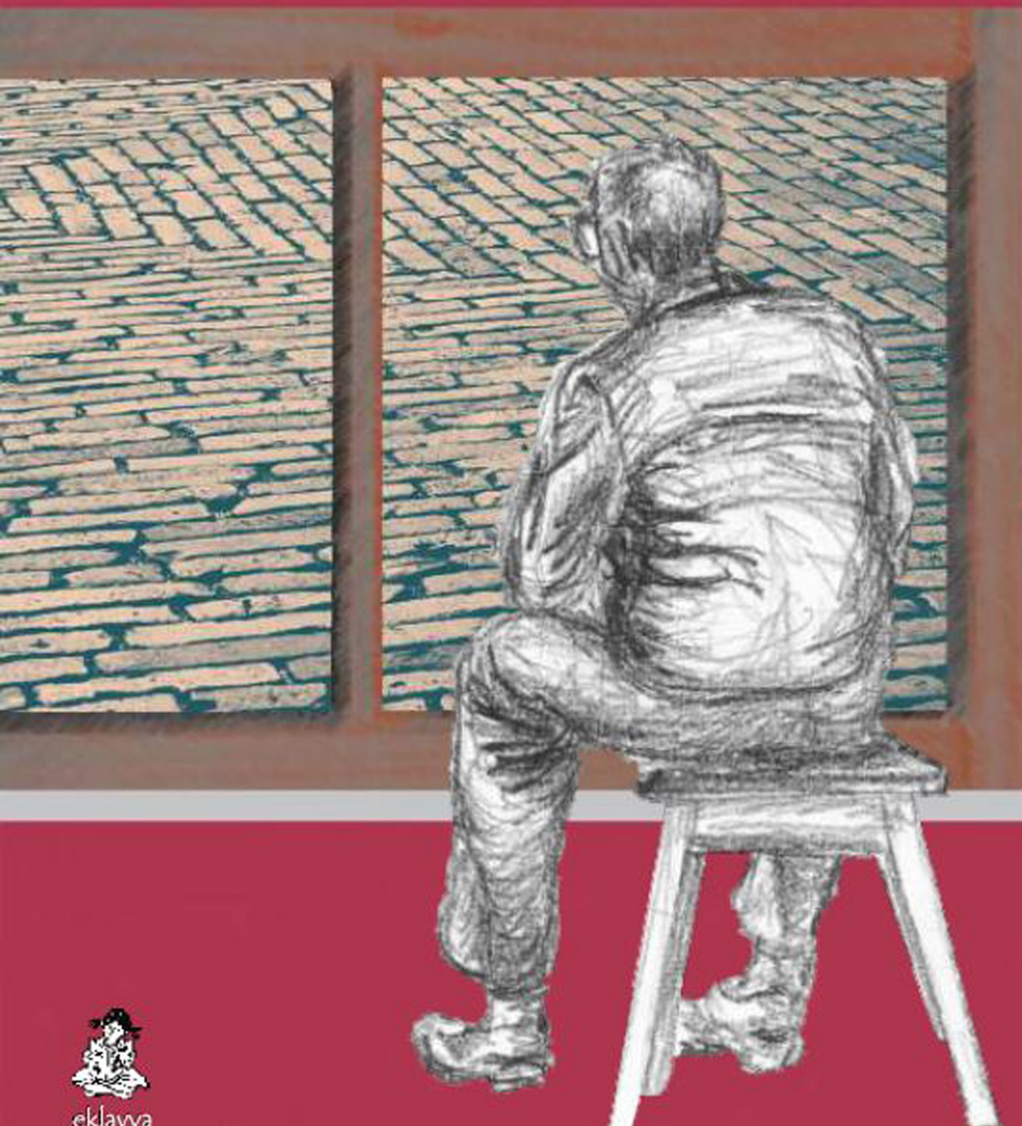
THE GENTLE MAN WHO TAUGHT INFINITY

SHESHAGIRI KM RAO



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Sheshagiri KM Rao (Giri) works as an Education Specialist with UNICEF in Chhattisgarh. Though he started out as a mechanical engineer in 1991, his keen interest in education led him to become a schoolteacher a few years later. Giri has worked with national and international organisations as well as with the government. His professional interests include early childhood development and school education, with a focus on the development of educational interventions in diverse contexts and the facilitation of organisational reflection, educational research and writing on contemporary educational issues. A special area of interest is that of chronicling people's narratives. His other interests include astronomy.

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Author: Sheshagiri KM Rao
Designer & Illustrator: Ishita Debnath Biswas

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“This book is dedicated to children, parents and teachers everywhere, with respect and affection… and in the belief that another education is possible.”

Foreword

There is much talk about education these days, mostly because the situation in most parts of the world is so dismal. Governments talk endlessly about the matter, trying out this scheme and that scheme. There is also much talk about teacher training, mostly empty talk because nothing ever seems to come out of it, other than more schemes. This is one area where East and West seem united! (We must note, however, that some countries have made significant breakthroughs in public education, notably Finland. But that is the subject of another book.)

A central issue is: from where do great teachers come? Can we manufacture them? We live in a self-help age, with countless manuals of all kinds available on the web: how to be a good parent, how to manage your business, how to be a good public speaker… Inevitably, this extends to school teaching as well: how to be a good teacher, how to be a good school principal, and so on. Continuing, we may ask, from where do great mathematics teachers come? This is a more challenging question! Mathematics continues to be a much-feared (indeed, hated) subject at the school level, despite the fact that so many exciting things have happened in the subject in recent decades that with good reason have been called “the golden decades of mathematics” - think of all the great breakthroughs made over the last three decades.

For the curious, I mention three examples: (i) the proof in 1994, by A Wiles, of Fermat's Last Theorem, the search for which had taken over three centuries; (ii) the discovery in

2002, by M Agrawal, N Saxena and N Kayal of the Indian Institute of Technology (IIT) Kanpur, of what is technically known as an efficient algorithm (simply, a set of rules) to determine whether a given number is prime or not; (iii) the proof in 2002, by P Mihailescu, of Catalan's Conjecture - the statement that 8 and 9 are the only two consecutive positive integers that happen to be perfect powers (namely, 8=2^3 and 9=3^2).

In an article by Stacy Zeiger on the Houston Chronicle's website, I found the attributes of a great mathematics teacher listed: (i) having an extensive knowledge of mathematics; (ii) having an awareness of the fact that students learn in different ways, using multiple strategies to help students grasp difficult concepts; (iii) providing students with the knowledge and tools to solve problems and encouraging them to solve challenging problems on their own; (iv) overall attitude and actions: a skilled mathematics teacher is respected not only for his or her knowledge of the subject, but also for his attitude towards other human beings and towards life in general; (v) caring about students: a great teacher focuses less on the content being taught than on the students being taught. It is not clear whether knowledge of these attributes will help us discover great teachers of mathematics! What seems clear is that despite all the obstacles that we human beings collectively throw in their way (and we are very good at this), great teachers of the subject do turn up here and there, apparently on their own. We must be grateful to life that such is the case.

It has become almost superfluous in today’s times to say that a good mathematics teacher – one who truly loves his or her subject – is a rarity. A good mathematics teacher who loves the subject to the extent that he keeps in touch with current developments by reading international journals and solving challenging problems from their problem sections (one as challenging as finding a proof of the Steiner-Lehmus theorem) is even more of a rarity. Add to all that a willingness to learn new things even at a ripe old age (such as learning a new programming language like C; anyone who has tried this will know what a challenge it is) and a rejection of corporal punishment during an era when it was considered not just normal but actually desirable, all the while operating in a school where regimentation and corporal punishment were 'part of the scene', and you have before you an individual who is exceptional, a once-in-a-generation teacher. Such it seems is the subject of this book.

I do not personally know Mr Channakeshava. I wish I did. Here is an individual who instinctively knows what education at its most essential level is all about, and mathematics education in particular: that it is about that wonderful attribute called play (not competitive play, not play that brings in prizes, not play that is to be graded, but play which exists in the moment, which has no purpose for its existence); that it is about beauty (that elusive quality which is so vital for us to glimpse and to touch, and one must realise at the same time that beauty is not a personal possession); about truth (and who does not recall here the lines of John Keats, 'Beauty is truth, truth beauty,' – that is all / Ye know on earth, and all ye need to know"); about love (the love of truth, of beauty, of elegant speech; the love of sharing; the love of watching another human being grow); and about justice and compassion. He knows that mathematics education is ultimately for human flourishing. (In listing these attributes, I am quoting from a beautiful address given by Prof Francis Su, President of the Mathematical Association of America; it was his outgoing address.)

We will probably never know how Mr Channakeshava came upon these powerful insights, but he has, and he lived these insights in the classroom, happily, without fanfare, without drawing attention to himself, which is surely the way that one should live. And for that, the thousands of pupils who have passed through his hands will be eternally grateful. It is highly significant that the ex-students who were interviewed during the writing of the book clearly remembered so many things about Mr Channakeshava so many years after they left school: his mannerisms, his style of delivery, his problem collections, and his way of proving theorems. Such is the impact of a great teacher.

Mr Giri too has done something unusual. As he writes in the “Author’s note” at the start: “Who discusses the teacher these days anyway? It is not a sexy topic. Very little is written about the teacher, especially the unknown one who quietly pegs away, lifting us from the morass we have sunk into. The media devotes more time to teachers who abuse children than to those who are doing great stuff.” This is all too true, and I applaud him for taking the trouble to write about a truly remarkable teacher. In documenting his story, he has held up a beacon in the fading light and has done a service to future generations of teachers as well as students. He is to be richly congratulated.

In closing, one may ask: what can one learn from Channa’s story? Is it at all possible to learn from the way he went about his teaching? Is it at all possible to institutionalise the kinds of things he did? Or is this a futile thought? These questions are difficult to answer. And yet, it is vital that we ponder on these issues. If we do, there may be a possibility of a true legacy to such a teacher.

Shailesh Shirali
April 2017

Shailesh Shirali is the Director of Sahyadri School (Krishnamurti Foundation India), Pune. He is also the Head of the Community Math Centre, Rishi Valley School (Krishnamurti Foundation India), Andhra Pradesh.

Author's note

Hello. My first book. How exciting!

First of all, a big 'thank you' for deciding to test the waters with me. I hope you will be rewarded for taking the risk. Before you plunge into your explorations, there are a few things about this book that you should know. Perhaps the journey will then be even more fun.

It would be apt to begin with this quote by the American historian, Henry Adams: "A teacher affects eternity; he can never tell where his influence stops."

My relationship with Mattur Venkatadri Channakeshava, the bespectacled gentleman who taught us maths in school, is a lot like that.

27 years after I graduated from Baldwin Boys' High School, Bengaluru, I decided to write about him. I felt this deep urge to reconnect with him. I wonder why it took me so long. After all, I must have passed by his house in Bengaluru many times during all these years. I could have met him as many times as I wanted to. I didn't. I guess it is all about waiting for the right time.

As Adams says, we end up picking up many things from our teachers in school, things that affect us in ways that we begin to understand only many years later. And what we pick up keeps visiting us time and again, in ways we cannot foresee.

Why am I writing about Channa?

The first sentiment that drives me is hope. Close on its heels is anger. I believe they are a potent combination.

I’m angry because our schools are letting down our children, day after day, year after year. Survey after survey by the government of the day, and those outside the government, show the abysmal depths we have managed to plumb. We cannot sink any further. The only way left is up. The crisis of learning is eating away at our vitals as a society. Just imagine – a child who reaches the eighth grade carries many years of 'learning deficits' in all the subjects she has tried to study in school. In effect, what she has managed to learn is only equal to what a third or fourth grader is expected to know. What a waste of her time! I’m not getting into a long-winded discussion about why this is happening. The undeniable fact is that this crisis is staring us in the face, challenging us to respond.

I remain hopeful, nonetheless, because I can tell Channa’s story. There is much in its telling that could make maths-learning enjoyable and open new vistas of beauty and discovery. Maths is a drudgery for many, given the robotic and mechanical ways of teaching the subject that we have perfected over the years.

Stories like Channa’s, I hope, will help us move out of these miserable depths. Out of the ‘mathsphobia’ that afflicts so many students. At least they may then look forward to school.

You are free to disagree with what I say, but I would still urge you to read on.

I was fortunate to learn high school mathematics from Channa. He didn’t make me a rank holder or a gold medallist in the subject. No, he didn’t. I was just an average student of the subject who very nearly failed in it in the ninth grade. He will vouch for this. But he made me love the subject. That is what he wanted from all of us in a school that was ultra-conventional and prided itself on exam results, a school where corporal punishment was the norm.

I want to tell you about the many things he did.

The first thing Channa did was make us enquire and think.

He gently nudged and pushed us to figure things out on our own, always. Second, his teaching was based on first principles… always. You never got a sense that he was ad hoc. He taught you to see a pattern in the madness. Third, he taught us mathematics because he wanted us to pursue truth and beauty, not just get good examination grades. Yes, you read me right. Never mind if none of us became mathematicians. For me, he made maths fascinating. Finally, Channa was a master storyteller who loved his craft. Which is why I remember minor details of his teaching even after all these many years.

Channa is the eternal student of mathematics who carries within his being a deep understanding of its structure, its mysteries and its paradoxes. These he happily shared with us.

So we stood on his shoulders and looked beyond, enjoying the mathematical landscapes that he unravelled and made visible. Beautiful, often startling landscapes that I hope to share with you.

My vantage point today is that of an itinerant wanderer in education. I contribute my two-paise worth to make schools and learning better. I have been at it for 24 years, in different places, trying different things, hoping they will make a difference.

We all have our theories about what ails the education system and what works well. Mine centres on the teacher. No surprises there. If teachers can be empowered to do better, a turnaround is surely possible. I realise that I'm making a very general statement here – teacher preparation actually needs nothing less than an overhaul. There are many things one can do to empower teachers. We have a vast treasure trove of government documents since independence – studies, commissions, reports and the like – that expound what needs to be done. I will not embark on that journey and I will not add to those documents.

Instead, I have chosen to write about an extraordinary teacher, his subject and his craft. I want to spread the word so that all of us – teachers, parents and those interested in education –

can learn a thing or two from him. Stuff that we can do and enjoy. And see the difference. I believe this is a positive thing to do.

Who discusses the teacher these days anyway? It is not a sexy topic. Very little is written about the teacher, especially the unknown one who quietly pegs away, lifting us from the morass we have sunk into. The media devotes more time to teachers who abuse children than to those who are doing great stuff. You are, of course, welcome to disagree.

I first wrote an article on Channakeshava in March 2012. It took me almost a month to end up with 15 pages. The missus, son and daughter sat patiently through this effort, wondering what I was up to after coming home from office. I was surprised that it got to 15 pages in the first place. As I began writing, the stuff that Channa had done all came back to me – one thing at a time. I suppose this is what great teaching does to you. It never gets out of your system.

I shared my article with a few friends. Some months later it was picked up by the National Council for Teacher Education for its online journal *Voices*. I felt happy to see the article on their website in the first half of 2013. Then there was a turning point when a suggestion was made that I could make the article more interactive and conversational. That is how I came to this point.

Now that we have gotten the bit about the background out of the way, let me tell you what you can expect to find in this book. It has three core elements intertwined in the pages that follow.

First, the book is about Channa, the maths teacher and person. He appears in all the chapters, except Chapter Zero, which is about my childhood, and Chapter Five, where I talk about Channa's influence on me when I became a teacher. In Chapter Six, I reflect a little on my journey in education. Channa, the person, appears later in the script. For instance, there are some interesting recollections of his persona and teaching by his students, from the 1960s to the late 1990s, when he retired from Baldwins. There are also my conversations with him in the very last chapter.

Second, I reflect throughout on Channa's pedagogy – the way he taught us maths and why I believe it is unique and important. I discuss the art of Channa's teaching.

The third aspect that I believe makes the book complete are the personal explorations on the nature of mathematics. I must say a little more about this. Some parts of these explorations are also in hindsight, especially the discussions on the notion of proof and the history of mathematics. So, I piggyback on Channa to talk a little more about mathematics itself. I cannot not do this, because the topics that Channa touched upon are fascinating and need to be shared with everyone.

The teacher, his craft and his subject – they are inseparable. These three elements are also built into the title of the book. I use the word 'infinity' in a metaphorical sense to show that the study of mathematics opens us to infinite imagination and possibility, enabling us to develop insights into the nature of the infinite.

Channa took us on a roller coaster ride into the world of mathematics. I say this deliberately. You will soon see why. Beyond the syllabus and our sterile textbooks, he showed us what mathematics is really all about – from cyclic numbers to Euclid to Gauss to Bhāskarāchārya, from Cantor to Euler and the Bridges of Königsberg, from the Barber's Paradox and the Four-Colour Problem to Fermat's Last Theorem. There was never a dull moment.

These intriguing problems, patterns, paradoxes and mysteries are at the heart of mathematics, he said. We students needed to understand them to truly appreciate and love the subject. The best part is how they are intimately linked to what we study in school. The saddest part is that they are not included in our syllabus. Our curriculum and syllabus makers have deprived us of the very best parts of mathematics, year after year.

Thanks to Channa, mathematics became a very human story for us, filled with meaning and a character of its own. He demonstrated how even the most complex mathematical ideas can be made simple for kids like us. I wanted to know more and more. What if I was just a so-called average maths guy?

That's the story I wish to narrate, not give a mere list of topics and leave it there. I could have simply said, "Channa talked about this and that and the other." That's what someone actually suggested – to keep it as a nice little article without going into the details. This book would never have been written then. After the *Voices* publication, I sent the article to another teachers' journal published by a well-known national-level educational institution. They liked it, but they too wanted me to bring it down to two pages, citing space constraints.

I understood their problem, but what set me thinking was their argument to abridge the article – that there was no point in talking about the problems, paradoxes and other conundrums in mathematics that Channa had treated us to, since these are now available at the click of a button on the World Wide Web. So they told me it was enough for me to mention that we were taught this and that, without getting into the details.

This argument seemed so convincing at first that I wondered for some time if the whole exercise of writing a book would be pointless. But then I realised that what I had done in the article was not a blind copying of content from different sources, including the Web. I was interpreting this content in the context of our classroom discussions, which was really the unique part of our maths-learning in high school. To remove these interesting descriptions and experiences would be to remove the soul of Channa's teaching. So I decided to expand, not abridge the article, even if it meant not getting published in a new journal of some repute.

Moreover, how do you search for something on the World Wide Web that you do not know even exists? This brings me to another problem that I'll just mention here but not discuss – the development enrichment materials for teachers. We have very little of such materials, especially in mathematics.

The three elements that make up this book jostle with each other in almost every chapter. Typically, the chapters begin with what Channa did in the classroom – how he introduced a topic, what stories he told us and so on. The actual examples are then discussed, to the extent that I could remember what had actually happened. Thereafter, driven by my own fascination for the subject, I have moved beyond to talk a little more about mathematics itself. I hope you like my meanderings on beauty in maths, Fermat's Last Theorem, the Barber's Paradox, the idea of infinity and controversies in the history of the subject.

The three elements that make up the book also need to be situated in the complex world of family, school, examinations, competitions, aspirations and the general malfunctioning of our education system. Whatever be our vision, something else invariably happens. So you will find my ruminations on this reality here and there.

You are a brave reader if you have survived thus far. One more thing – the maths need not frighten you because it's all school-level stuff. I have tried to keep things simple and have explained as much as possible. Allow yourself to be surprised. Stretch your imagination a bit. For the discerning reader, there is more stuff to mull over in the additional notes. This section contains the more advanced material. If you feel the need to go even further, I hope 'Channa's 20' will satiate your appetite.

Finally, I must tell you whom this book is intended for. The ones who first come to mind are those who were hassled by mathematics as students and now as parents – who are trying valiantly to make sense of all the maths mumbo-jumbo thrown at them. Then come the stressed teachers who try hard to make their students understand maths and get good grades. And, finally, the book is for just about anyone who is interested in the education of children. I'm sure that's a huge number of people.

So, let go of your mathsphobia. There is no examination to be passed. You are warmly invited to leap into the fray. What else could be more important?

Giri
Raipur
July 2017

My heart beating fast, I alight from the taxi in front of his house. A sense of anxiety and foreboding seizes me. Will he be here, I wonder. 1985, the year I graduated from school, was 27 years ago. My only consolation is that I had met him 12 years ago in 2000, when I had invited him to my wedding. He hadn't made it though.

Other teachers have passed on – Devadas, the English and geography teacher; Verghese, the English teacher; JT Williams, the music teacher; Suputra, the biology teacher; Mohan Murthy, the other high school maths teacher who taught Section A… the list seems endless. Channakeshava? He just has to be around and I'm desperate to meet him. Many years after I moved on from school I wrote about him, and now want to know more. I want him to see what I've written. I've connected again with him after so long through my writing. I now want to see him in flesh and blood. Will he be here to see me?

In a bit of a daze, I knock on the door of 'Vasuda', his home in a place called Banashankari in south Bengaluru. Long moments pass in trepidation before his wife opens the door. She looks at me quizzically and asks, "Yes?"

"Can I meet Mr Channakeshava?"

"Yes. And you are…?"

I mention my name. "I was his student in Baldwins in the 1980s."

She invites me in, perhaps wondering why I have come.

He is sitting on the floor of the living room, watching a game of cricket - it was the Indian Premier League season. Phew! What a relief to see him in flesh and blood again. He is surprised to see me, I guess. But Channa, I remember, is not very expressive, except when he is teaching maths.

We start talking…

0 Zero: in the beginning

“The maths you did at school is not all of it. Better still: the maths you didn’t do at school is interesting. In fact, a lot of it is fun.”
– Ian Stewart

In the beginning, there was… well, there was me, growing up and all in a house in a small lane named after someone called Muniswamy. I’m not sure if you have heard of Muniswamy Road, a small lane off Queen’s Road in the cantonment area of Bengaluru. A quiet, nice place it was, my Bengaluru.

I was an only child and my parents, granny and I lived in granny’s ancestral home at No 14, Muniswamy Road. It was what my father described as an ‘outhouse’. There was this big plot of land at the end of the road, perhaps 50 feet by 100 feet, and our home was situated at the back end of the plot, leaving a big open area in the front. There, we had the tulsi katte, which my mother and granny worshipped every day. Nearer the gate, on either side of it, were two big trees – a sampige mara, whose yellow flowers smelt heavenly, and a coconut tree that my mother said had been planted the day I was born. There were other flowering plants and some shrubs in our compound that had a rather tall wall built of stone. I played with my friends and rode my tricycle in that open space in the front.

Anasuya, my mother, whom I lovingly called amma, and Janaki, my granny, whom I called aaji, looked after our home while Madhusudhan Rao, my father, my anna, worked in the Indian Telephone Industries to earn a livelihood. ITI, as it was known in those days, was then perhaps the only company in India that manufactured telephones. If you wanted to call someone living outside Bengaluru, you had to book a ‘trunk call' that could take a few hours to connect. That was the pace at which things worked those days. That world was different.

Today, we call our telephone connections ‘landlines’ to distinguish them from the mobile phones that have invaded our lives. These contraptions weren’t even part of our wildest dreams in our youth. The World Wide Web was also beyond the realm of our imagination. I spent my childhood in a simpler world, unfettered by digital technology, with no anxiety to be eternally ‘in touch’, playing with my friends in the by-lanes of Muniswamy Road, climbing the sampige tree with its divine-smelling flowers and cycling in our compound.

Anna would not settle for anything less than a church-run school for my education. For him, such a school was the epitome of educational quality. The emphasis on teaching in English, a language seen by many as a passport to success in the world, and the ‘discipline’ that he thought was part of the ethos of every church-managed school, would help me go forward in life. He was, I remember, fascinated by all things Western – people, places, gadgets and lifestyles. I suppose he believed that Western culture was in many ways superior to the culture of our poor country.

“In Singapore,” anna would tell us, “you can actually sit on the road and eat your food, like we do. Their roads shine and their buildings touch the sky.” What he meant was that the roads in Singapore were clean, unlike in our city. I would try to imagine what this city looked like, with its huge buildings. The only comparable building we had in Bengaluru those days was the Utility Building on Mahatma Gandhi Road, which came up sometime in the 1970s. Though anna did not have the resources to travel to the West, he had quite a few penfriends from different parts of the world. In particular, I remember Benny J Klinger who sent me coins and stamps from the USA. I wonder where Benny is now.

I'm not sure what my mother or granny made of anna's arguments or beliefs regarding the selection of my school. They didn't seem to be too happy. But even if they did have opinions, it did not make any difference to anna's outlook. I remember heated discussions at home about the excessive fees of 56 rupees per month that was paid in 1978 to educate me in Baldwin Boys' High School. By the time I came to grade 10, the fees had increased to 78 rupees per month.

"How will we run the house if you put him in such a school?" amma and aaji confronted anna routinely.

But anna was adamant. He was prepared to scrounge from his meagre salary to give me what he thought was a good education. "He has to go to a good school," was all he said every time this difficult topic came up. He would then raise his voice to silence everyone. An uneasy calm would reign after that.

The first school I attended was St Anne's Convent on Cunningham Road, a couple of kilometres away from our cantonment residence in the much quieter and more beautiful Bengaluru of those days. The city has lost its character now and is unbearably polluted and noisy, with its vicious traffic jams. It is no longer the 'air conditioned' or 'garden' city or the 'pensioner's paradise' that it was made out to be in the days gone by. As the buildings and people pile up, Bengaluru increasingly becomes a stifling 'concrete' experience with flyovers, 'fly-unders' and now the metro – all desperate measures for desperate times. We need more fundamental solutions for our urban problems.

People seem to have migrated to Bengaluru from around the country. One reason is because the city acquired the status of India's 'Silicon Valley' in the 1990s. This attracted hundreds of thousands of people to the IT industry that was being set up at the time. The salubrious weather, quiet roads and friendly people also acted as incentives for the migrants to stay on and settle in the city.

I used to go to school in a cycle rickshaw. An ageing rickshaw-puller with thick glasses and an unkempt beard took four or five of us every morning to St Anne's Convent. We would get off the rickshaw on Queen's Road and push it up the incline till we reached Cunningham Road. Our rickshaw-puller didn't have the strength to pull us up Queen's gentle incline. But on the way back, it was an easy roll down for him. Still, we would jump off and run behind the rickshaw, much to his irritation.

In 2001, I took my missus Savitha one evening to 14, Muniswamy Road to see what it looked like. For a while, I couldn't locate the house.

"Are you sure we have come to the right place?" Savitha asked with rising impatience and a hint of sarcasm.

"Here it is!" A few minutes later, I pointed out to a huge concrete structure in the corner, opposite our neighbour Gowda's house. 14, Muniswamy Road had changed beyond recognition.

Samiullah, the man who had bought our house in 1979 for just 79,000 rupees, had taken utmost care not to leave even an inch of the plot uncovered. Gone was the sampige, gone was the tulsi katte, gone was my coconut tree and gone were the many plants that had given us flowers and made our outhouse beautiful. In their place was a monstrous concrete structure, occupying every bit of 14, Muniswamy Road, my childhood home. It felt as if I had been trampled upon.

We were routinely caned at St Anne's. It is surprising how quickly one gets inured to caning, which sometimes got brutal on our knuckles, ears and butts. Apart from caning, our teachers discovered another unusual way to punish us boys. We would be made to sit between two girls for at least one period – as if this imprisonment would be embarrassing enough to make us mend our ways! I wonder what the girls thought about this insult to their dignity. They looked ill at ease anyway. And we were all too innocent to look at it in any other way.

I don't remember getting excited about maths at St Anne's. But I was quite agitated about my Hindi. I just couldn't get my little fingers to write those funny-looking letters in the Devanagari script. So my mother would unhappily do my homework. During every Hindi class, I would quietly slink away to an adjoining section to escape the wrath of the Hindi teacher, returning only after the period was over. In those days at St Anne's, we had more than 50 students in a class, so I don't think the teachers noticed a new face, or were even bothered if they did.

When I reached grade 3, anna must have been worried about my next school as St. Anne's did not allow boys after grade 3. Since he had set his sights on the well-known Baldwin Boys' High School located near Johnson Market on Hosur Road, I was coached every evening to clear the entrance test.

"You'd better pass the test," warned amma. "Anna is spending all his money on you."

"It just flashed in my mind that there is this Baldwins and then there is Bishop Cottons School," I remember anna saying. He must have cycled past these schools a thousand times. Both had formidable reputations and anna was impressed by how they looked, with their big playgrounds and old colonial-style stone buildings. His mind was made up as he started dreaming for his son. He was saving his hard-earned money to get me into one of these schools.

I joined the 98-year-old Baldwin Boys' High School in 1978. The school, one of the oldest in Bengaluru, was managed by the Episcopal Methodist Church of southern India. It had an interesting history. It was established in 1880 by an American philanthropist from Connecticut called John Baldwin, who ploughed the profits he made as a businessman into establishing educational institutions that were open to all children regardless of their colour or gender.

Baldwin's views on education ran counter to the conventions of his time, even in his own country. In 1880, on learning that European children in India were not getting a decent education, he managed a decent grant of some 3,000 dollars to purchase the land on which Baldwins stands today. There was no looking back after that. Over time, the school also opened its doors to Indian children.

For reasons beyond my comprehension, I had to repeat third grade despite completing it at St Anne's. Maybe the chaps who admitted me to Baldwins thought that my third grade certificate from St Anne's wasn't good enough. They made me lose a year. But nobody seemed to be bothered about it. Anyway, anna was happy that I had made it to the school of his dreams.

Baldwins was a pretty regimented school. I'm sure you know what I mean. Most of us have grown up in such schools. We sat in rows and columns, and got caned as if it was our birthright. Sometimes, our books were flung across the classroom if we did not do our homework. In addition to caning, some teachers would punish us by making us stand on the bench for an entire period, towering over the others. This was an amusing experience. We had to show respect for our teachers, even if we cursed them in our minds or behind their backs, or even if we were afraid of them.

Baldwins was a school that sorted you in class through the usual tests, exams and the like. But I do believe our teachers cared for us. They wanted us to do well in school and in life. The intention was right but the method surely was not.

Channa was different.

I did well enough to be picked up for the junior and senior cricket teams. But I must add that we had a lot of politics over team selection. There was heartburn. I was also part of the school choir till grade 9. We had a great music teacher in JT Williams. They said he could play 15 different musical instruments. In high school, I did a lot of debating. In the National Cadet Corps, I became a corporal and sported two stripes. Thus the school gave us ample opportunities for things other than studies.

In 1980, when I was in the sixth grade, we celebrated 100 years of Baldwins. I remember the entire year went in preparing for the centenary celebrations, which lasted three full days. Baldwin Girls' High School also took part in the centenary. For the first time, I felt attracted to the girls who came regularly to our school to prepare for the great event. Why didn't we study together? The story goes that boys and girls started studying separately around 1900 for reasons unknown; Baldwin Girls' was established then.

There was nothing unique about my mathematical experiences up to the seventh grade. We had sincere teachers, all women who went through the motions of teaching the syllabus and preparing us for the exams. I don't recall being excited about maths. Not that things would have been different had we been taught by men. But the school had this policy of appointing women teachers for the lower grades. Till middle school, we had women teachers for almost all the subjects.

Mrs Thomas, our class teacher in sixth grade who also taught us maths, was perhaps the best of the lot. She was always pleasant, never lost her patience, and once told me she could solve all the problems in the fat and complicated-looking tenth standard ICSE (Indian Certificate of Secondary Education) textbook written by a chap called OP Sinhal. My respect for Mrs Thomas went up several notches after that statement of hers, even if I did not have the chance to verify the claim. I saw my seniors lugging the book to school every day and shuddered at the thought of having anything to do with it. That complicated-looking book containing strange symbols aroused both fascination and fear in me, and if me Mrs Thomas could solve every problem in it, I could not help but admire her for that.

I have memories of two ‘mathematical’ events, both to do with my incomprehension in grades 6 and 7. The first was to do with the idea of inequalities. Since anna was confident of teaching me till things got mathematically tough for him around grade 7, he took it upon himself to teach me those vexed inequalities using signs like >, <, ≤, ≥ which appeared in between two or three numbers. Something was greater than or equal to something, while it was lesser than or equal to something else. Or so it went, endlessly.

These symbols can tie you up in knots, leaving you confused and bitter. One evening, when much of it went over my head, anna asked me to do what he called guddipaatam. “Mug it up!” he said rather harshly, when he saw me struggling with some problems. He was really cross with me that evening. I was startled to see him that way, for he is really gentle. “I can’t!” I shouted back at him, upset that I was so dumb. He must have been stretched to breaking point then.

Guddipaatam was his constant message addressed to me throughout my school and college days, even when I went for my engineering course many years later. If you don’t understand something, ‘mug’ it up – memorise it – and vomit it in the exams. I don’t think much has changed in our schools even today.

I couldn’t figure out the concept of inequalities despite anna’s best efforts that evening. My reward for incomprehension was a hard slap. That was the only time he slapped me. “What have you done to my lenkroo?” aaji cried when she saw me sobbing. I could see that anna was ashamed, apologetic and emotional about what he had done. For a few minutes, he cursed himself by banging his palm hard against his forehead. I sobbed without control. Suddenly, everything felt so miserable. But we got over it pretty quickly.

The second event in my maths-learning that I remember took place in grade 7. My numerical world was shattered when integers came into it. It was for the first time that the ‘number line’ appeared in my life. I found the idea of integers, those negative numbers that extend the other way from zero on the number line, difficult to understand. How could you have a number that was less than zero? Till then, I thought that zero was the lowest you could go (also, the lowest you could get in an exam). Zero basically meant nothing. How could one think of something lower than nothing?

Why did we need these numbers in the first place? The whole idea seemed ridiculous. Suddenly, one had to be mindful of the ‘sign’ of the number when one added, subtracted, multiplied or divided numbers, using all sorts of brackets. What puzzled me was how a negative number multiplied by another negative number could yield a positive number. How could that be?

Till then, everything seemed fine in mathematics. One had to ‘solve’ problems, get the right answers and get good marks. Or so I had thought. I was good at this and usually got away with top grades. But integers changed all that. The inequalities had also taken their toll and I remember disliking profit-and-loss problems in arithmetic as well. I could never get them right.

Anna gave up teaching me mathematics around grade 7. The stuff we were learning was beyond him. So he took to teaching me English and social studies, where he would make elaborate notes in his neat handwriting. Basically, what he would do was answer all the questions in the exercises in our text book (by copying the answers from the book) and then give them to me on a platter. He thought he was making life easy for me but he was actually doing me a disservice by not allowing me to figure things out on my own. I guess parental anxiety makes you do these things. Many parents have little patience with their children.

Little did I realise as I went through the motions in school during those early years that mathematics would hold a world full of surprises for me in the form of strange creatures (numbers) with peculiar names such as rational, irrational, surreal, transcendental, perfect, imaginary and complex, which were related to each other on the number line and had even stranger relationships with each other in the number world. Mathematicians often describe these relationships between numbers as 'beautiful'. Notice a sense of the mystical in the names of some of these numbers.

Cough.

Did you hear me say 'beauty'? That is what we started seeing in the summer of 1982.

Enter Channa.

I was always in Section B – from grade 1 till grade 16 when I graduated as an engineer. Had I been in Section A, I would not have been Channakeshava's student. This book would never have been written.

Talk of happy coincidences… this had to be the best of them.

If a story has to be written, the universe will conspire to get it written. Here I am, writing it.

1 Beauty

"Why are numbers beautiful? It's like asking why Beethoven's Ninth Symphony is beautiful. If you don't see why, someone can't tell you. I know numbers are beautiful. If they aren't beautiful, nothing is."
– Paul Erdos

I reached the eighth grade in 1982, just as the monsoon set in. Section B. By then, we had shifted to our new house in a locality called Koramangala. I had begun cycling to school on my brand new red Hero cycle. It was Channakeshava's first class with us in June that year and it had very little to do with our syllabus and textbook. He could have so easily started off with the first chapter in the textbook and we would have plodded on as usual. Or he could have given us some revision sums and sat and watched us struggle with them. That would have been the easier way out.

Instead, he did something very different, which no other teacher had ever done before in any of my classes. He began by showing us 'beauty' in mathematics. I say this in hindsight because it wasn't obvious then. I'd never seen beauty in mathematics until then even in my wildest dreams. What happened in the period that day was quite astonishing even as it was fun.

What I do remember is Channa walking into our classroom on the ground floor of the 23-year-old stone building that we called Lincoln Hall, dressed immaculately in a dark-coloured suit. The class fell silent. I don’t remember the formalities. He must have said something to start his relationship with us – a curt greeting and a measured smile, perhaps. A few moments later, Channa began by posing the following question on the blackboard:

142857 x 1 =

That was easy as you can very well imagine. Next, he asked:

What about 142857 x 2?

While we got busy with the task, wondering why he was testing our multiplication skills and what was coming our way next, I saw him quietly writing the answer on the board: 285714.

Then, he asked again:

And 142857 x 3 would be…?

428571 is the answer, which he had written down as if he knew it all along, while we were busy multiplying!

“Sir, how can you write the answers even without multiplying?” someone from behind me, who was as perplexed as I was, threw this question. But Channa was unfazed, choosing not to say anything. Quietly, he went on to show what happens when 142857 is multiplied by 4, 5 and 6. Interesting stuff was emerging:

142857 x 1 = 142857

142857 x 2 = 2857<u>14</u>

142857 x 3 = 4285<u>71</u>

142857 x 4 = <u>57</u>1428

142857 x 5 = <u>7</u>14285

142857 x 6 = <u>857</u>142

What bounced off the pages of our notebooks was a ‘cyclic permutation’ of the original number’s digits as we multiplied it by 2 till 6. It was difficult to discern any pattern in the way the digits got shifted – sometimes one digit got shifted, sometimes two, and sometimes even three, but in all cases the ones getting shifted were consecutive digits. What was startling was that it was the same number 142857 whose digits got recycled and shuffled around, like cards in a pack. Oh boy! How could that be?

Such numbers are called ‘cyclic numbers’, Channa told us, waiting for his statement to sink in. *“Mathematics is full of such curiosities, which can be studied by just about anyone,”* he added. For example, we can ask – how many cyclic numbers are there? Is it possible to discover all of them? Is there a formula that can help us find them all? When we learn mathematics, we need to often ask such questions. They lead us to discover truths and patterns underlying things happening in our world. Like cyclic numbers, there are other numbers with even more fascinating properties.

We continued exploring 142857. Channa asked, *“What happens when we multiply 142857 by 7?”*

We wondered if the original number would get recycled again in some way. But no! What we got instead was:

142857 x 7 = 999999

This was puzzling, indeed! Can you figure it out? Actually, if you express 1/7 (also known as the ‘reciprocal’ of 7) as a decimal you get the recurring pattern 0.142857. And when 0.142857 is multiplied by 7, you get 0.999999. Therefore, when we multiplied 142857 by 7, we got 999999.

Well, I couldn’t figure out the logic. It looked mysterious and magical at the time. Sometimes, I believe this sense of the mysterious should persist. It keeps us engaged. Understanding, which is essential when we learn, removes the mystery. But it also opens up more questions. So, the sense of the unknown and the mysterious keeps persisting even as we keep understanding more. It is actually a never-ending journey.

While we were soaking it all in, Channa just looked around and smiled. If every maths class was going to be like this, I thought, it's going to be a lot of fun! Channa's first session with us was quite a shift, used as we were to drill and more drill in the earlier grades. Why weren't we shown these things before?

And that is how I was introduced to the beauty of mathematics in June, 1982. I use the word 'beauty' deliberately because it hits you like a beautiful sunrise does. Everything appears to be in place and it cannot get better than that moment. You are left speechless, maybe even thoughtless. That is how it should be if you want to fully savour a sunrise and immerse yourself in it.

"So, how was school today?" amma asked this question almost every day.

"We have this new class teacher who also teaches us maths. He is really good…"

That evening, I told amma whatever little I had made out about my new teacher Channa. She just smiled. But later she would keep asking once in a while, "How is your Channa?" And an amused aaji would sometimes make up her own song using the word Channa with some other rhyming word.

I'm not sure how my friends reacted to what Channa said and did. I guess they were just as intrigued. I must say it is very difficult, if not impossible, to figure out what an individual child will find attractive in a maths class. What excites me need not have a similar impact on my friends. But it is the task of the teacher to always engage students in exploration. Who knows when the spark will be lit? For me, it was lit in a classroom in Lincoln Hall 35 years ago.

Let me take this discussion further because there are important lessons for us to reflect upon, which arise from what Channa showed us. In mathematics, beauty lies in the patterns that unfold. This need not be restricted to numbers alone. Geometrical patterns are equally fascinating. Students of mathematics can access and explore this beauty, marvelling at it and asking questions about why these patterns emerge. Teachers themselves have to go on the same journey with their students for this to happen. I was fortunate to have had Channa as a teacher.

I believe it is the responsibility of every maths teacher to help every student see this world of mathematical beauty. For that to happen, we need to go beyond the notion of teaching as a rigmarole, a drill exclusively aimed at getting good marks in an examination. Unfortunately, much of what we learn in school is geared towards passing exams and getting good jobs. Should these be the only tasks of education? I believe we must move beyond this narrow, utilitarian view. Teaching and learning mathematics must introduce the student to a world of beauty and fun.

"What about my teaching time?" you might anxiously ask, if you are a teacher who is racing against time to 'complete the syllabus'. My response, very simply, is that your teaching time will be more effective because you will be motivating students to become more receptive to maths-learning.

Let's now go back to our cyclic number example. What happens when we multiply 142857 by 8?

This is what it looks like:

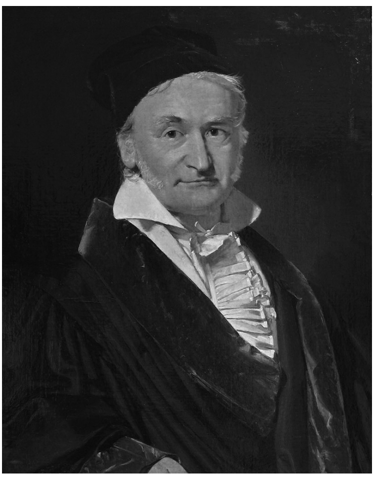
142857 x 8 = 1142856

Is there anything about the number on the right hand side that strikes you as interesting?

There are other cyclic numbers like 142857 that behave in pretty much the same manner. One of them is

0.0588235294117647 (do 1/17 on your calculator), and the other is 0.052631578947368421 (do 1/19 on your calculator). Where are they coming from? How many more of these numbers exist? Can we find out?

You might now begin to think that this cyclic pattern has something to do with the reciprocals of prime numbers (7, 17 and 19 are prime numbers, as you can see). But when you find out the reciprocal of 1/13, where 13 is also a prime number, you get 0.076923. This is not a cyclic number. This is how one looks for relationships and patterns in the world of numbers. In essence, this is what Channa wanted us to do in his very first class. He was coaxing us to look at our maths-learning differently. He was asking us to look for underlying patterns.



Gauss (1777-1855 CE), the prince of mathematics

With Channa, the teaching was clear and cogent. Maths-learning was almost effortless because we began to see things with more clarity. And then, there were the stories and anecdotes.

A few months later, Channa shared one of his astonishing stories. *"There was this great German mathematician called Carl Friedrich Gauss. Do you know what he did when he was in school?"* We stopped fidgeting and listened. "*The story goes that when Gauss was in school in the 1780s, all of 10 years old, his teacher gave the class a problem to solve:*

Find the total of 1 + 2 + 3 + 4 +....+ 100"

I guess Gauss's teacher wanted to laze around that day. There is sometimes this great temptation to just give students some task and relax.

While the other boys in his class were sweating this one out, tediously adding each number, Gauss already had the answer. As was the custom of the day, Channa said, he placed his slate on his teacher's table. The other slates took their time in coming. What Gauss did was this: He rearranged the numbers 1 to 100 in such a way that he got 50 pairs of numbers, each pair totalling 101. The problem then became simply 101 x 50 = 5050. Wow! Isn't this remarkable ingenuity, that too for a 10-year-old?

When Channa showed us what Gauss was said to have done then, I was astounded. This is how it looks:

1	2	3	4	5	6...	47	48	49	50
100	99	98	97	96	95...	54	53	52	51

So, we have 100+1=101; 50+51=101; 95+6=101 and so on. Each pair adds up to 101, and there are 50 such pairs. Therefore, 50 x 101 = 5050.

Isn't this beautiful? I wonder how Gauss's teacher reacted. For me, it was an example that completely disrupted my notion of learning maths. It turned my opinions upside down. I was compelled to look at this monotonous subject afresh. It was not about getting the correct answers. It was about the way we looked at things. It was about appreciating the deeper structures and meanings of mathematics. We could do it with our teacher and there were many rewards waiting to be enjoyed.

I'm tempted to share a few more examples of the beauty of numbers. I include them just to illustrate all that is possible if we start looking for underlying patterns.

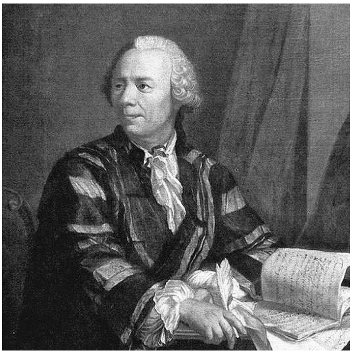
$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = ?$

$13 + 15 + 17 + 19 = ?$

$21 + 23 + 25 + 27 + 29 = ?$



Leonhard Euler (1707-83 CE) is often regarded as the most prolific mathematician in history

Do you see this pattern? What could it mean? Does it work with even numbers as well? We can ask many questions about what happens next in the series.

This is just a random example. There can be many more. The point I wish to make is that there is more to our number system than meets the eye. It has many unusual and often startling features. Some of these are just quirks, but many have a deeper meaning. Discovering these beautiful features can be a very rewarding experience. Mathematicians often spend their entire lives looking for them. This is how the subject develops.

Children need a little coaxing, as I have discovered. But once they set out, the journey can only be fun. It really doesn't matter if you are studying or teaching in a school that focuses almost exclusively on examination results. Once a beginning is made, there is no looking back. This is what Channa wanted us to believe – that we all have the seed of mathematical ability, and we can use it pretty much the way Gauss did.

I would be doing injustice to this discussion if I do not mention the most famous example of mathematical beauty, as seen by mathematicians themselves. It is the Euler (pronounced 'Oiler', not 'Yooler') equation, after the great Swiss mathematician, Leonhard Euler. This is what it looks like: $e^{i\pi} + 1 = 0$.

Wait! There is nothing to be alarmed about! Each of these numbers 'e', '*i*', 'π' and '0' is unique and they keep recurring in mathematics. I have discussed this fascinating equation in some detail in the additional notes given towards the end of the book. The physicist and Nobel Laureate Richard Phillips Feynman once referred to it as: "… the most remarkable and almost astounding formula in mathematics." I suppose he said this because some of these numbers with their non-terminating and non-recurring decimal places (as in the case of 'e' and 'π', whose decimal digits just keep going on and on and on…) combine in strange ways in the formula to yield zero. That is what stumps me. How does it happen?

The cyclic number pattern that Channa shared with us, and the couple of examples I have mentioned above also remind me of alchemy. It is very much like what I once saw sitting in the chemistry lab of our school. Wilson, our stern-looking chemistry teacher, who instilled fear but was nonetheless considered by many as a 'solid' teacher, was demonstrating how different liquids react with each other. There was this colourful liquid which was mixed with another colourful liquid and voila! The mixture became colourless! There was a collective gasp in the room, and since fairly large glass containers were used to pour out these liquids the entire exercise looked spectacular. For once, the stern Wilson smiled. Of course, we did not know enough of physics or chemistry then to say why this colour change had happened.

The notion of beauty in mathematics must be important, indeed. The 2014 Fields Medal in mathematics (the equivalent of the Nobel Prize, which is not awarded for mathematics) went to a Canadian-American of Indian origin called Manjul Bhargava. The International Mathematics Union, in its information sheet on his work, has described Manjul thus:

> *"A mathematician of extraordinary creativity, he has a taste for simple problems of timeless beauty, which he has solved by developing elegant and powerful new methods that offer deep insights."*

Elaborating on his work, Bhargava has said:

> *"Intellectual stimulation, beautiful structure, applications – elliptic curves have it all."*

'Elliptic curves' are a certain kind of curve that one comes across in maths, but they are certainly not the elliptical orbits that describe the Earth's path around the Sun. But let us not bother about elliptic curves at this point. Instead, the question we need to ask ourselves is: Why are we not getting children to see and explore this beauty in mathematics? I believe this can happen at a young age and with all children. That is when they begin to see that maths is not just calculations. This subject is anyway not about calculations, just as literature is not about typing.

Since we will keep talking about schools and education, and since schools have different purposes, we shall explore the role of mathematical knowledge, including the knowledge of numbers, in the education of children. All these discussions about number patterns and beauty bring me to the question: Why care about numbers? This question then leads to: Why care about maths at all?

So let me take this discussion further, beyond the realms of mathematical beauty. One way of looking at the study of numbers (and hence, the study of mathematics), as a student or teacher, or just as someone who is interested in the subject, is to treat it as an intellectual pastime having no connection whatsoever with the real world. But that is not actually the case, as we have so often discovered. Eventually, in some way or other, these seemingly abstract patterns actually tell us a lot about how the physical world in which we live is designed or ordered.

I'm not just referring to how mathematics is used in daily life, say when you go to the grocer's, or when you design an aeroplane. While mathematics is surely at play here, I'm going much further to state that the mathematical patterns and relationships that we discover in our minds are intimately linked to the structure of stars, planets, atoms, cells and their behaviour. They explain the way the physical world is made.

Now, that's a fascinating idea worth exploring. It is something that I realised more and more during my explorations in physics. To me, it seems as though there is a deep relationship between our consciousness (which enables us to do things like mathematics) and the structure and behaviour of the natural world. Physicist and Nobel laureate Eugene Wigner terms this relationship as the: "... unreasonable effectiveness of mathematics in the natural sciences." He elaborates why he thinks it is unreasonable:

> "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful... that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement."

Wigner wonders how mathematics, which occurs in our minds, can actually help us understand how the planets move, apart from enabling us to understand and predict so many natural phenomena that happen all around. If we take this line of thinking further, I suppose it will not need much convincing to show that there is something useful in mathematics.

However, there is a breed of humans that does what is called 'pure' or 'genuine' mathematics because they claim that the pursuit of is abstract and does not have anything to do whatsoever with the real world. The famous mathematician GH Hardy, who had invited the self-taught genius Srinivasa Ramanujan over a hundred years ago to Cambridge to collaborate with him, viewed the links between mathematics and the real world with utter disdain. He even asserted that these pursuits were inferior to the pursuit of mathematics for its own sake:

> "All this is in its way very comforting to mathematicians, but it is hardly possible for a genuine mathematician to be content with it. Any genuine mathematician must feel that it is not on these crude achievements that the real case of mathematics

rests, that the popular reputation of mathematics is based largely on ignorance and confusion."

But the abstract does link up with the real. The abstract number patterns of mathematics tell us something about how our minds work and how nature is structured. They can even help us unravel how our society works. They can throw light on questions such as: Why are some people rich and many poor? How do some people amass so much wealth? How do the material conditions of the rich and poor compare? How does the government use the money collected from tax payers? How do some people stash their money away in tax havens?

These questions may seem disparate. But mathematics – more specifically, the knowledge of numbers – can be used to teach and appreciate issues of social justice. The objective of learning mathematics then becomes broader, perhaps more urgent, if we can employ it to expose and fight injustice and oppression in our world. So it gets further and further away from what Hardy thought. But I must leave it here, for it will need a separate discussion. I shall come back to it later.

Whichever way you are inclined to see the learning of numbers – you can take your pick from the viewpoints I have described – Channa cast a spell on all of us with his cyclic number example and the number magic of Gauss during the monsoon of 1982. With that one master stroke, he showed us that learning mathematics is not just about revising last year's problems when you get promoted to the next grade, nor is it about mechanical problem-solving of exercises at the end of each chapter in your maths textbook. It is certainly not just about preparing for your term-tests and exams to score good marks. What Channa was saying was: "*Look! Your maths-learning is all about discovering patterns and relationships. It is about embarking on a great adventure into an unknown world, full of surprises and happy rewards*."

I believe the notion of beauty and the search for patterns is central to any mathematical adventure. Hence, it is central to mathematics education as well. This is what we should learn from Channa. It is the first thing I would like to draw your attention to.

It could be pointed out that the cyclic number example may after all be just a quirk of the number system and that there are many other examples that more substantively illustrate the idea of mathematical beauty. I would beg to differ. If the number 142857 and Gauss's shortcut have remained with me even after 35 years, then there was something about those experiences that left such a lasting impact on me as a student. And if there are more such numbers, I'm not so sure they are mere quirks of the number system. Surely there is something going on here, something deep.

I must also point out that people look at numbers in many other ways, as I have realised over the years. They play a role in poetry, in matters related to heaven and hell, in astrology, and in the numerous religions and cultures of humanity. Our discussions didn't get there though.

There wasn't much exploration of the world of numbers after that. It had to be business as usual in a school that prided itself on examination results and that provided little space for its teachers to be imaginative. But there was a definite difference in the way we were taught mathematics. Something had changed in me. Forever.

More was to come.

There was this unmistakable gleam in Channa's eyes whenever he conveyed something fundamental or profound. I remember waiting impatiently for his classes where he would tell us some new story or the other.

When he emphasised some deeper aspect of the topic at hand, his mouth would open a bit wider than usual and he would pull his lips back to make a point – like when we first heard him use the Latin phrase 'Quod Erat Demonstrandum' (QED) while we were grappling with the idea of proof in geometry.

I remember I laughed when I first heard Channa use the term, almost quoting it like a Sanskrit sloka. He had surveyed the amused looks on our faces. QED literally means, "What was required to be proved." It is what one says when one has done all the hard work, using reasoning to demonstrate the proof of some statement. A proof is a kind of achievement in the mathematical world by which one demonstrates underlying patterns and relationships between numbers, spaces and the like.

Everyone was attentive in Channakeshava's class, from the first to the last bench, because we knew we would end up learning something. There were no discipline issues and Channa never once raised his voice or banged the table to get our attention.

2 QED: the pursuit of truth

"Whence do we derive our knowledge? From observation. But we can observe only individual cases... from instances we have observed. We feel warranted in concluding that what we found true in those instances holds in all similar ones, past, present, and future, however numerous they may be."
– John Stuart Mill

"How do you know that the three angles of a triangle add up to 180^0? How do you know it is true?" Channa asked us with a straight face one day. This was in grade 8 after the first term. The question seemed simple and the answer obvious. But I could detect a mischievous intent as he posed the question. Little did I know that he was about to take us on a journey that was more than 2,000 years old, beginning with a chap called Euclid.

We hadencountered the idea of a 'proof' along with the term 'theorem' for the first time when we were learning geometry, sometime in the middle of 1982. Till then, we had learnt some 'basics' in geometry – how to construct different kinds of triangles, find out the missing angles, how to bisect angles and lines, and so on.

Somewhere along the way, we had taken for granted the most common property of triangles – that the sum of the three angles of a triangle always adds up to 180^0. This was sacrosanct.

I had always wondered how one had arrived at this number 180. Why not 246^0, for instance? Anyway, I didn't ask this question then and it took me years to realise why we choose 360^0 for a complete angle, 180^0 for a straight angle and so on. I now believe it is largely a matter of convenience, for the number 360 has a large number of factors that make computation easier.

"How do you know this is true?" Channa persisted.

"Measure and see, and you will get 180^0," many of us promptly responded. I remember being surprised by the question. It seemed so obvious then! And the protractors in our compass boxes were anyway made to show 180^0. So how could we get anything else? The matter was therefore closed in my mind.

*"**How many** triangles should I draw and measure?"* Channa kept on.

This question stumped us a bit and I remember that we didn't agree on any one number. In fact, any number would have been arbitrary – 10, 50, 100, 1000… The class fell silent after a while. Channa had a point and we were unable to get around it with our argument to collect data for as many triangles as we could. But I kept wondering: Could triangle number 1001 be different if the angles of each of the first 1000 triangles added up to 180^0?

That was, indeed, his next question, *"What if the angles of the 1001st triangle do not add up to 180^0?"* There was no response from the class. What was he trying to get at?

To drive home Channa's point a bit, I must share another example, called the 'Monstrous Counter Example' that I came across recently. This thing called mathematics can be very unforgiving, as this example tellingly illustrates. Consider the statement: "The expression $\{1+1141n^2\}$, where 'n' is a natural number, *never* gives a square number." A square number is a number like 25, because it can be written as 5x5, where 5 is called the 'square root' of 25. The term 'square' is used because 25 can also be represented geometrically as a square of 5 units by 5 units. You can think of several such square numbers, which are simply called squares.

When computers were used to check this expression, people found out that it did not yield a square number for any natural number from 1 till 30,693,385,322,765,657,197,397,20 7. This latter number is of the order of septillions, not millions, billions or trillions. Anyone could have then concluded that the expression $\{1+1141n^2\}$ will never yield a square number for all n. But – and this is downright crazy – the expression gave a square number for the next natural number! Can you figure out the square root then? Difficult to believe, right? Quite astounding, in fact. That is how the number world can startle you.

I remember Channa saying, *"For this reason, we have to prove that no matter what, the three angles of a triangle add up to 180^0."* So we went about proving this elementary theorem and learnt along the way that the word 'theorem' is nothing but a statement claiming such and such a thing, which has a proof that is generated using what is called 'deductive reasoning'.

In mathematics this is something like saying, "If A = B and B = C, then A = C." In common parlance, it is like saying, "All apples are fruits, all fruits grow on trees; therefore, all apples grow on trees." In more intricate cases, each step of a proof has to lead to the next one in a logical manner, till you reach a conclusion. You can make your own examples of deductive reasoning. We do it everyday, though we do not always recognise it. Sounds pretty straightforward, doesn't it? But it needn't always be the case. For some theorems, the proofs can run into hundreds of pages, like the proof of what is famously known as 'Fermat's Last Theorem', which is nearly 150 pages long! It was called the 'proof of the twentieth century'. More on that later.

For me, this was a new way of doing mathematics, used as we were till then to mainly doing calculations of various kinds. This was a new kind of animal that had to be understood. It wasn't easy. But it was an important step we were taking in establishing *truth* or *falsehood* in mathematics.

That was not all. Channa kept gesticulating repeatedly in an arc with his right hand, something he did whenever he had something serious to say. *"All these theorems in geometry that one encounters in school – and breaks one's head against – rest on certain foundational statements called axioms or postulates,"* he said. In simpler terms, axioms are fundamental assumptions that look obvious. We also say they are self-evident.

Euclid (circa 400-300 BCE) causes a headache for students even today. This sketch is illustrative because we do not know what Euclid looked like.

"The geometry we were learning at that moment," Channa said, *"was founded on a few assumptions that a chap called Euclid made more than 2,000 years ago in Alexandria in Greece."* Phew! It is called 'Euclidean' geometry and is written up in a series of books titled *Elements*. In fact, for almost 2,000 years, there was only Euclidean geometry.

There is a lot of controversy about Euclid. Some historians even wonder if someone named Euclid ever existed! Other Greek mathematicians often do not even use Euclid's name and just refer to the 'author of the *Elements*'.

"Without axioms or assumptions," Channa told us, *"all the geometry we were learning would collapse. It would not make sense and there would be utter chaos. These assumptions are starting points without which we cannot move forward to prove a statement."* Things were getting quite intriguing!

I just sat and listened, trying to make sense of what he was saying. We hadn't heard such stories in mathematics before.

In fact, we had never heard any stories before we reached the eighth grade. We just used to 'solve' problems. Our mathematical lives had been simple, but boring.

Just in case you are wondering what this is all about, let me spell out Euclid's five axioms. Then we shall try and see how, without these assumptions, we would land up in trouble. "Let the following be postulated," said good old Euclid in the *Elements*:

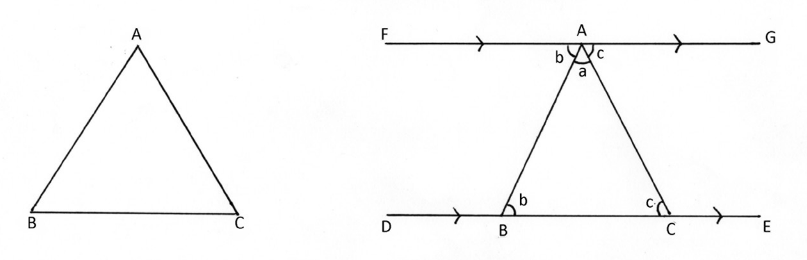
1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one end-point as centre.
4. All right angles are equal to one another.
5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles (This is famously, and controversially, known as the 'parallel postulate').

I remember being perplexed – why state the obvious? As if echoing my thoughts, Channa said that these assumptions are, well, assumptions at best. They cannot be questioned. They have to be accepted as they are. But they arise from everyday experiences and from the reality of our lives. And they provide us invaluable starting points, as we shall soon see. Euclid was beginning to give me a headache. What were we getting into?

Channa gave us another example. He said that this was a bit like the game of chess. The queen can move back and forth, horizontally and across, while the rook can only move horizontally and vertically. We do not question this. It is an assumption or rule that is fundamental to the game. If you change this, the game changes and you perhaps get another kind of chess. So the rules are self-evident and we don't question them. Axioms are like that.

The best way to appreciate the usefulness of Euclid's axioms is to consider the example of the theorem that states the sum of the three angles of any triangle (say triangle ABC in Figure 2.1(i), often written as ΔABC) is 180^0. This is usually the first theorem we all get to prove in school.

The Triangle Sum Theorem



(i) (ii)

Figure 2.1

A common way of proving the theorem is to do some constructions involving ΔABC as shown in Figure 2.1. This is normal practice. We are permitted these constructions as long as we do not change the form of the original figure, which in this case is ΔABC. What we have done here, as Channa did in 1982, is the following:

1. Through point A, we drew a line parallel to the opposite side of the triangle (BC). We marked two points F and G on this parallel line. Remember Euclid's parallel postulate? We will bring it into use here.

2. We also extended the side BC in both directions and marked two points D and E on this extended line. It is clear from Figure 2.1 that lines FG and DE are parallel

to each other, that is, they will never meet even if they are extended indefinitely on both ends.

Before we got this far, we had already learnt that when two parallel lines are intersected by a third line called the transversal, the 'alternate' angles formed are equal to each other. This is illustrated below in Figure 2.2. The two parallel lines are FG and DE, and AB is the transversal that intersects these two lines.

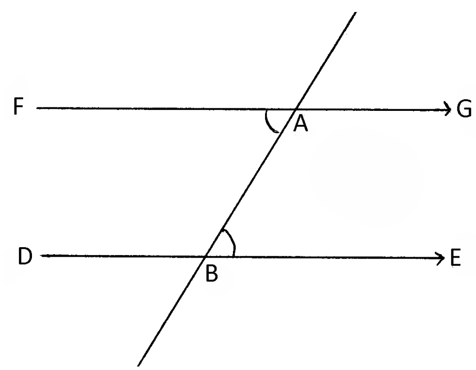


Figure 2.2

We can have any number of transversals cutting across these two parallel lines. The angles FAB and ABE are the alternate angles which are equal.

We can now go back to Figure 2.1 to identify the other set of alternate angles that are equal. The parallel lines FG and DE in this figure intersect two transversals, line AB and line AC. It follows that:

Angle GAC = angle ACB (since they are alternate angles.)

Similarly, angle FAB = angle ABC (since they are also alternate angles.)

Now, angle BAC (marked by the letter ‘a’ in Figure 2.2) is the third angle on the straight line FG. You will remember from your school days that all the angles on a straight line add up to 180^0 – which is known as a ‘straight angle’ and is half the complete angle of 360^0. Hence, angle GAC + angle BAC + angle FAB = 180^0.

But angle GAC = angle ACB (which is one of the angles of the triangle) and angle FAB = angle ABC (which is also one of the angles of the triangle), as we have already seen. The third angle BAC (marked as ‘a’) is common to both the straight angle as well as ΔABC. We can, therefore, say that the three angles of the triangle ABC add up to 180^0.

“QED!” Channa promptly added after finishing his argument, waiting for it to sink in.

Proofs can be awkward, elegant, beautiful, stunning… Mathematicians often use such adjectives to describe what they feel about these logical processes. I am not a mathematician, but I too feel that this most elementary proof is elegant. It is also unnerving because this kind of reasoning takes time to get used to. But it is at the heart of mathematics, whether you are dealing with geometry, algebra or any other topic.

“Alright, but what is happening here?” you might ask. How is all of this related to those grand, but obvious axioms that we started with? So let us go back to them. We can actually see how the axioms are used in this proof. In fact, without the parallel postulate, we cannot even prove that the angles of a triangle add up to 180^0. No, we can’t.

Let me now play the devil’s advocate and have an imaginary conversation with Channa. I go up to Channa and say, “Sir, I don’t like these axioms. They are so confusing.”

As is his wont, he smiles. But there is a quizzical expression on his face. *“What’s the matter, ra?”* ‘Ra’ is a word from Telugu which is used to affectionately address a boy. Channa often used it.

I continue, “Suppose, for the sake of argument, we do away with a few postulates. Is it still possible to mount a strong proof of the ‘Triangle Theorem’ that we have just proved? I have a problem accepting it as something self-evident and beyond questioning.”

This axiom business, I remember, got on my nerves.

Channa immediately sees where I’m coming from. He knows how to deal with it. *“Alright,”* he says, *“let’s eliminate axioms 1 and 2 since you do not like them anyway. What happens then?"*

> *“If you eliminate axiom 1, you cannot join any one point with any other point using a straight line! Axiom 1 provides us that foundation. And similarly, if you take away axiom 2, you cannot extend the side BC of the triangle ABC on both sides, as we have done. Got it?”*

Then I think, if we also remove axiom 5 (the parallel postulate), we cannot use the equality of the alternate angles when a transversal intersects two lines that are parallel to each other. Thus, it would be impossible to mount a robust proof that the three angles of a triangle add up to 180^0.

That’s it then. We cannot live without these axioms. I suppose you need a set of assumptions to develop any logical system of thought. Euclid’s geometry is one such system of thought that we study in school.

At a slightly more intricate level, Euclid’s axioms can tell us something about the nature of the space in which we live. When I say space, I mean the physical reality of our lives. We take it for granted and do not question it much – such as, its structure. I vividly remember this discussion with Channa. There were some arguments, too. For instance, if we take the second postulate of extending a line in either direction, we arrive at the idea that space is ‘unbounded’. Who actually knows if this is so? One can have endless arguments about whether the line can go on forever. Then there is the controversy of postulate 5, which I shall come to in a minute.

Our example here is one from geometry. And it led me to accept why axioms are important, critical even, before we started deducing certain statements or propositions like the Triangle Theorem or countless other theorems in mathematics. Euclid's brilliance, perhaps, lies in his explicit articulation. Historians of mathematics often point out that Euclid's results were reported by earlier mathematicians in different cultures, given that geometry itself arose from the practical need for measurement in different societies. But Euclid, they say, was the first mathematician to develop a comprehensive system or rules for logical deduction. These rules marked the transition from concrete examples to abstract concepts, an approach that informs high school maths education today.

I'm recounting all of this because Channa shared this fascinating history with us. Imagine – we were learning something that was thought of two millennia ago and put down in a book then. Little did we imagine that mathematics could be like this. In the coming months and years, we would prove many theorems, in geometry, algebra, trigonometry and algebraic geometry, all of which are part of the regular diet of maths of every high school-going student.

None of this was part of our textbook. Channa need not have touched upon it. But he did, believing that all of us were worthy of being told this story. That is why he took the trouble to tell it to us. For a parent or teacher, this conviction is important – that the child needs to, and can develop a deeper understanding of the subject. Unfortunately, we often underestimate children and undermine their potential. We write them off and then struggle to get them past tests and exams. Some of us even go further with our biases. We believe that certain children coming from certain communities cannot learn. Some teachers also believe that boys learn better than girls. How then can we treat all children equally?

I liked the study of theorems and their intricate proofs, but only intermittently, and kept wishing that Channa would tell us more and more stories instead. There were days when the proof of a theorem would appear effortlessly. In particular, I remember the flash of insight I had when we were discussing the Triangle Midpoint Theorem. There was this surprised look on Channa's face when I supplied the arguments of the proof with conviction. That was the first time I had managed to mount a process of reasoning. I felt empowered.

But there were also days when I would struggle. Channa would, however, go on as enthusiastically as ever, adorning blackboard space with these eternal theorems and their mind-bending logical proofs, questioning us at every step. He never failed to write QED after every proof was done. Writing QED after each theorem was a ritual that he religiously followed.

Many years later, much to my interest, I learnt that Mahatma Gandhi appreciated the power of reasoning in Euclid's *Elements*, which he was taught in school. In his autobiography, *The Story of My Experiments with Truth*, he reminisced:

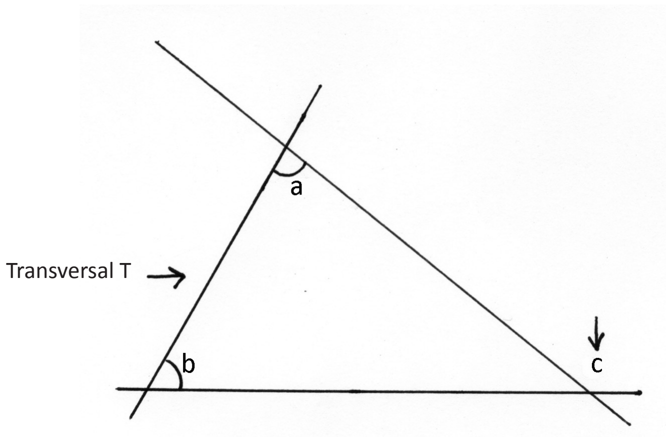
> "When I reached the thirteenth proposition, the utter simplicity of the subject was suddenly revealed to me. A subject that required pure and simple use of one's reasoning powers could not be difficult. Ever since that time, geometry has been both easy and interesting for me."

For those who continue to struggle with Euclidean geometry in the initial months of school, Gandhi offered consolation as if by saying: "Wait for the thirteenth proposition, I'm sure things will change."

The first theorem that we proved with Channa represented a real shift in my understanding of mathematics. There was something different about this experience during which we were gently nudged by Channa to move beyond what is sometimes called ‘naïve empiricism’. That phrase simply means that you cannot build your trust in patterns by plugging in some numbers to see if the same results hold for a certain number of cases – like when we told Channa that if for a certain number of triangles the sum of their internal angles adds up to 180^0, then this should be so for all triangles.

This is of course the method used in science, where we deal with the real world of objects. We rely on empirical measurements based on our observations of the natural world. From these observations, we build a general theory about why something is happening. The theory gets strengthened with more and more similar observations. Then, at some stage, it becomes a law. But in the triangle example, we got stumped because we did not know how many triangles we would need to measure in order to establish the truth of the theorem. It was similar in the more complicated case of the Monstrous Counter Example. Then we encountered Euclid who showed us a way to prove that this theorem holds, no matter how many triangles’ angles we measure. So, mathematical proof is an indispensable tool to establish that a claim is true.

But there is a twist to the tale! Let me now come to the controversial parallel postulate, which leads one to conclude that space is flat, not curved, since those two parallel lines can keep going on indefinitely. This view had to be modified with the emergence of ‘non-Euclidean’ geometries nearly 2,000 years after Euclid. I can only touch upon these geometries here. Channa, too, discussed them briefly.



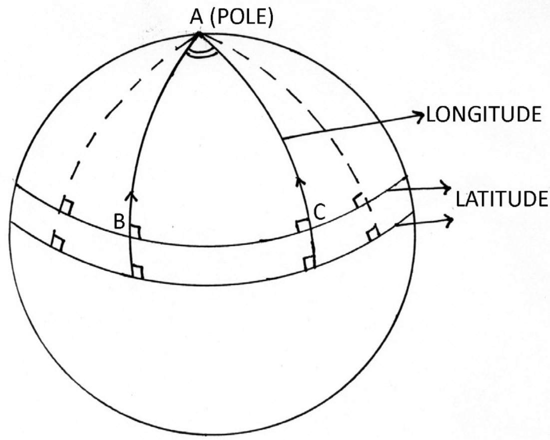
When a transversal T cuts two lines that are not parallel and converge at point O, the sum of interior angles a and b is < 180º.

Figure 2.3

Recall the essence of what Euclid said in his axiom 5: If two lines intersect a third in such a way that the sum of the inner angles ‘a’ and ‘b’ is less than two right angles (180^0), then these two lines, if extended far enough on the side where the angles are formed, will inevitably meet or intersect. This is shown in Figure 2.3, where the sum of the interior angles a + b is < 180^0.

On the other hand, according to Euclid, if the inner angles a + b = 180^0, they will never meet, however much one extends them.

Now, look at Figure 2.4. It presents a situation that turns this understanding on its head.



Sum of the angles of ΔABC is > 180°

Figure 2.4

This sketch of the Earth shows the way the network of latitudes and longitudes are constructed (recall your geography classes and check this out on a globe). The two longitudes (shown by arrow marks) intersect the latitudes (also shown by arrow marks) at right angles. Hence, the two longitudes are parallel to each other according to the parallel postulate. The postulate says that these lines will never meet if they are extended indefinitely. But in this case they meet at the poles, forming a neat spherical triangle ABC on the Earth's surface. So here we have a clear case of a triangle the sum of whose angles (A + B + C) is > 180^0! This calls for a different kind of geometry in which the parallel axiom of Euclid simply doesn't work.

So Euclid isn't enough. Got that? There is more. Triangles need not always be drawn on flat surfaces. We can draw them on curved surfaces as well. Another interesting aspect which can be seen in the diagram is that the latitudes are also parallel to each other. But if we were to take a walk or a train ride along any latitude, say the Equator, we actually come back to the point we started from. This is not possible in a Euclidean space where parallel lines extend in both directions indefinitely.

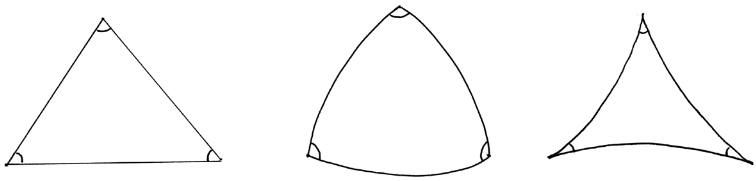
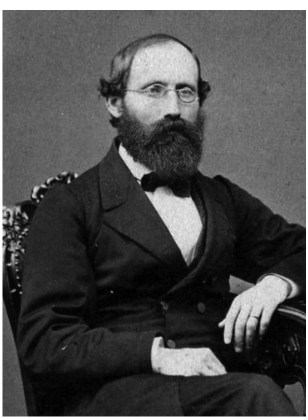


Figure 2.5

We can take this line of discussion even further. Look at the three triangles in Figure 2.5, which define three kinds of spaces: a flat space (consider the top of your desk in class) in which the sum of the three angles of a triangle is equal to 180^0 (this is what we are introduced to in school), a spherical space (with a positive curvature, like that of Earth's surface) where the sum of the three angles of a triangle is more than 180^0, and a hyperbolic space (with a negative curvature, like the surface of a horse's saddle) where the sum of the three angles of a triangle is less than 180^0. The geometries of these latter two surfaces are quite different from the one we learn in school, since the parallel postulate does not hold true here.

The two most well-known non-Euclidean geometries are the 'Riemannian' and 'Lobachevskian' geometries.



Bernhard Riemann (1826-66 CE) and Nikolai Lobachevsky (1792-1856 CE) challenged us to think beyond plane Euclidean surfaces and showed that there could be equally consistent geometries on other surfaces.

I sometimes wonder why it took so long for mathematicians to discover non-Euclidean geometries when artists and sculptors, for instance, had already been working on curved surfaces. Then there are curved mirrors. It requires considerable skills as well as understanding of the properties of curved surfaces to work on such areas. But let me leave it there for now.

You can also visualise these three kinds of spaces with a clay pot (the familiar *matka*) or a vase, which is an object on whose surface you can draw all the three triangles shown above, depending on which part of the *matka* or vase you draw them on. Think about it.

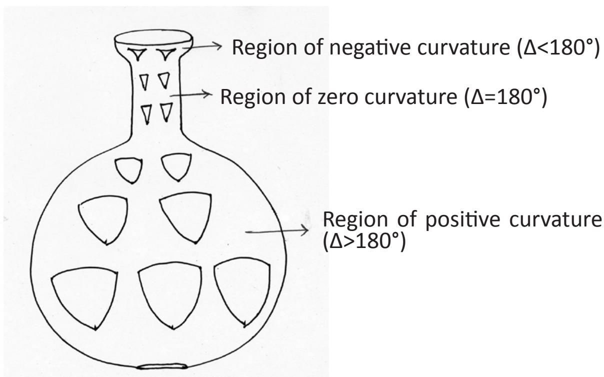


Figure 2.6

Let us get back to our discussion on the idea of a proof. One more thing strikes me as very important as I write this. Grappling with the idea of proof in high school may seem burdensome and there are some who might argue that it should not be taught to children as young as 13 or 14 years of age. But to me, a proof in general, and in mathematics in particular, is nothing but the pursuit of truth. It is the systematic search for and a conclusion about underlying patterns and relationships.

In my opinion, the pursuit of truth is the central task of education that goes beyond helping a student to prepare for passing exams and getting good grades. The question here is:

How do I know if a thing is true? And then, how do I know what I know? Here I need to be sure of the methods I use to establish the truth or falsity of any claim I'm making. These are important questions for education. I believe the pursuit of truth is not just a question of acquiring true knowledge but also living it in action. This will of course need a separate discussion.

The message I got from Channa is about this pursuit. He wasn't someone who asked us to just do problems and move on. At various carefully chosen points, he would share perspectives that went way beyond what our syllabi demanded. He opened up many windows that allowed us to peep inside multiple worlds, inside the boundless patterns and potentials of our minds.

To me, the development of perspective is important in learning. It goes beyond getting good grades. Perspective is what makes you engage deeply with a subject and, ultimately, develop a love for it. In the process of learning, developing perspective means knowing why you are learning something. It is also about knowing how the idea or concept you are learning developed over time. And most of all, it is about figuring out how what you learn is linked to so many other pieces of knowledge and the myriad aspects of our lives. Perspective is what liberates us as learners and often makes us exclaim "Aha!" This brings us joy.

Stories like the ones Channa narrated helped me develop perspective. They enabled me to understand where I was located in my learning of mathematical ideas. I now realise that there can be different kinds of truths, depending on our 'frame of reference' – from where and how we see things. We have already seen this in the case of Euclid, though it took nearly 2,000 years for new frames of reference to emerge. What is considered true in Euclid's geometry is not true in another kind of geometry. Knowledge itself is not absolute.

We were well and truly getting into the thick of things.

Aristotle, one of the great thinkers from antiquity, once said, “If you would understand anything, observe its beginning and its development.”

Channa used this as a fundamental principle in his teaching. History, coupled with story, was used to telling effect.

We were all in the before. We went there again just to figure out where we needed to go next.

3 History: of what use is it?

"I'm sure that no subject loses more than mathematics by any attempt to dissociate it from its history."
– Glaishier

Imagine a classroom of ninth graders who are about to learn the topic of 'logarithms'. Imagine a teacher who walks in and simply states: "Today, we will learn about logarithms." Looking a little unsure as blank faces stare at him, he turns to the blackboard and starts writing, perhaps equally unsurely:

> If $a^x = y$, then x is called the logarithm of y to base a, which is written as $x = \log_a y$. So, if $2^3 = 8$, then 3 is called the logarithm of 8 to base 2, which is written as $\log_2 8 = 3$.

The teacher then turns to face you. What would it have been like had you been a student in his class? Don't bother just yet if you do not know what this creature called 'logarithms' is. We will come to that later. Logarithm is actually a very ingenious idea if explained well. Otherwise, it can become quite a nightmare, literally a bitter logjam.

I can sense helplessness creeping in. To be fair, not all teachers would start a class on logarithms in this manner. More likely, they will entertain some questions from the students. If a student asks what logarithms are all about and why they are needed, the teacher might say they are needed to simplify complicated arithmetic calculations. If the teacher has heard and read about John Napier, she might mention him as the person who discovered logarithms. She may then parade some 'basic' (but crazy-looking) formulae like:

$$(\log_x ab = \log_x a + \log_x b) \text{ and } (\log_x a/b = \log_x a - \log_x b)$$

I hope you are still around. Again, don't bother about what these formulae mean, at least not just yet.

Before long, you will see these students moving to the sums at the end of the chapter, wondering what has hit them. You can bet they too are asking, "What is this all about? Why do we need this? Who is responsible for this?"

Each of these questions has an interesting story, which can greatly enliven what happens in the classroom. Teachers need to be encouraged to access such stories, which help to cultivate curiosity and make learning fun. Maths classes can end up as drudgery if this build-up is not in place. We then lose a great opportunity to infuse context and meaning into the maths that is taught.

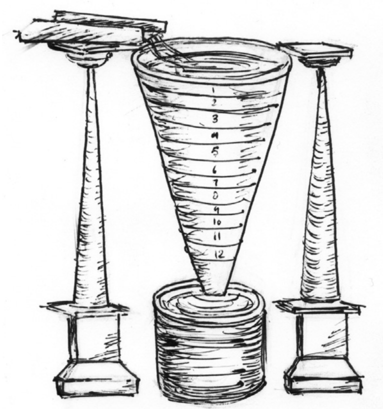
I will tell you the story of logarithms a little later. I can assure you that it will be nothing short of a revelation. For now, let me continue sharing the stories we heard from Channa. We need to start seeing what they mean to us. But let me do that after the story I'm about to share.

In our second year with Channa, when we were learning to solve quadratic equations, he told us the story of the Indian mathematician Bhāskarāchārya II and his daughter Lilavati. Channa had picked up this story from the translation of Bhāskara's *Lilavati*, a book that his friend Venkatram had given him during his childhood days in the by-lanes of a beautiful village called Mattur, located in the district of Shimoga in Karnataka.

Bhāskarāchārya II (1114-85 CE) is often referred to as medieval India's greatest mathematician. This sketch is only illustrative as we do not know what he looked like.

We learnt from Channa about an unhappy Lilavati who, the astrologers predicted, would never marry. But Bhāskara, being an astrologer himself, was able to calculate a date and time when she could marry. If that moment passed, her marriage was out of the question. The mathematician then designed a clock that would fill up with water and then sink at exactly the time Lilavati could marry. But Lilavati, in her excitement and impatience, leaned over this clock and did not notice the pearl that fell into it from her necklace, blocking the orifice through which water was slowly filling up the water clock.

Thus, the auspicious moment or *muhurtha* could not be pinpointed because the mechanism of the water clock had been disrupted. Lilavati was left unhappy and forlorn. She did marry eventually, but her husband died soon after. His untimely death was attributed to the inauspicious time of their marriage. To console his heartbroken daughter, Bhāskara wrote *Lilavati* (a treatise on arithmetic) sometime around 1150 CE. This, he believed, would immortalise her name. Well, that objective was certainly achieved.



An artist's sketch of the water clock.

Figure 3.1

This story both amused and intrigued me. There were so many things happening in mathematics, going beyond the sterile quadratic equations. I imagined the life and legend of this great mathematician and his daughter somewhere in south India sometime around 1150 CE. What must those times have been like? I did not even know till then that there were Indian mathematicians who had tackled problems like this. At that time, we thought that all the mathematicians in history came from Europe or thereabouts. Many of us still think so, forgetting that India had a rich tradition of mathematical thinking going back a few thousand years. That itself was a big learning for me, a realisation that came only in Channa's maths class. In our history periods we would have merely glossed over this 'piece of information' as something to be 'mugged up'.

As I gradually came out of my reverie, I saw Channa neatly write out the following problem on the board in his impeccable handwriting:

"In a lake surrounded by chakra and krouncha birds, one unit of length of the top of a lotus stalk is visible above the surface of water. On being swayed by a gentle breeze, its tip gets submerged three units away from its original position. Quickly tell me the depth of the water."

This is a very liberal English translation. The actual Sanskrit version, which is in verse form, reads like this:

chakra krounchaakulita salile kwaapi drishtam thadage
thoyaadurdhwam kamala-kalikaagram vithasthipramaanam
mandam mandam chalitamanilenaahatam hastayugme
tasmin magnam ganaka kathaya kshipram ambhah pramaanam

This is how it looks in Sanskrit:

चक्र क्रौंचाकुलित सलिलै क्वापि दृष्टं तडागे
तोयादूर्ध्वं कमल-कलिकाग्रं वितस्तिप्रमाणं |
मन्दं मन्दं चलितमनिलैनाहतं हस्तयुग्मे
तस्मिन्मग्नं गणक कथय क्षिप्रमम्भःप्रमाणं ||

This was one of Channa's favourite *Lilavati* problems that he posed to his students year after year. Quite a mouthful, isn't it? For those of us who don't know Sanskrit, it sounds like mumbo-jumbo.

But this was the language used by Indian mathematicians in those times. Mathematicians like Bhāskara were also poets of a high order who composed their mathematics in verse form. Aesthetics was integral to their work. The *Lilavati* problem is also beautifully and aesthetically conceived, and Channa drew it as skillfully on the board to help us analyse it.

Suppose we say that the length of the submerged part of the lotus stalk as given in the figure is 'x', then the total length will be (x+1) units. The length that is submerged represents the depth of the lake at the position where the lotus stands. If we assume that it is an upright lotus, it means that it makes a right angle with the surface of the water. We also assume that the lotus is rooted in the lake bed. (Lotus plants actually float but let's excuse Bhāskara for this indulgence – his oversight is not that serious.)

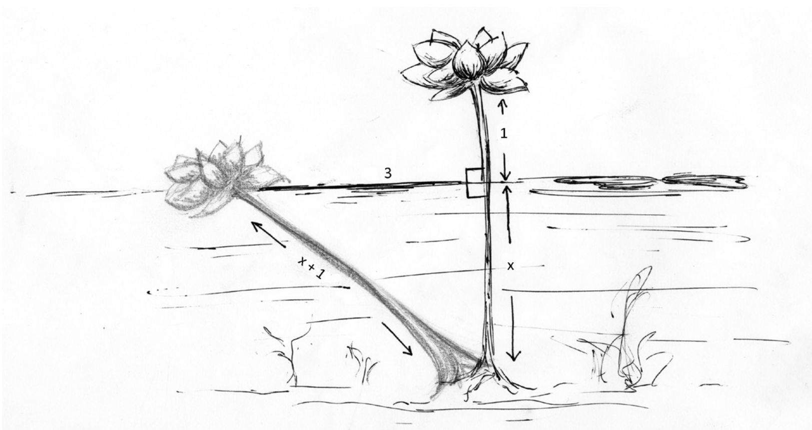


Figure 3.2

In this problem, (x+1) units is the length of the stalk that is swayed by the breeze to settle 3 units away from its original position. As our illustration shows, we now have a right-angled triangle whose sides are 3 units, x units and (x+1) units. (x+1) represents the hypotenuse (the entire stalk that is now submerged), while 3 and x are the other two sides. Applying the Pythagoras Theorem, we get the following:

$(\text{Hypotenuse})^2$ = sum of the squares of the other two sides

$(x+1)^2 = x^2 + 3^2$

On expanding the left hand side and simplifying the equation:

$x^2 + 2x + 1 = x^2 + 3^2$

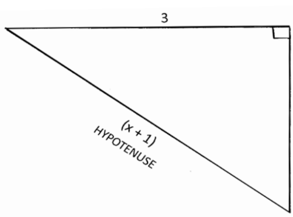
$2x + 1 = 9$

$2x = 8$

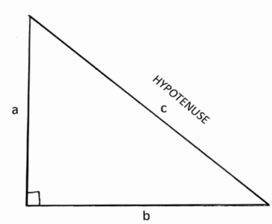
Therefore, $x = 4$ units, which is the depth of the lake at the point where the stalk is rooted on the lake bed. “Behold!” Bhāskara must have exclaimed, much like Channa's Euclidean habit of saying, “QED”.

I must make one more point about this equation. It starts off as a quadratic equation (because the highest power of x is 2, as in x^2) but ends up getting solved as a simple equation because the x^2 on both sides cancel out. Many problems in the *Lilavati* are quadratic in nature. They x^2 do not cancel out like in this problem but need to be solved using a method called factorisation, which students learn in high school. I have included one such imaginatively described problem towards the end of the book, just in case you need to whet your appetite.

In the event that you have forgotten good old Pythagoras Theorem, let me refresh your memory with the illustration of the right-angled triangle (Figure 3.3) that represents the *Lilavati* problem, comparing it with any right-angled triangle having sides a, b and c.



Therefore, $(x+1)^2 = x^2 + 32$



$a^2 + b^2 = c^2$

Figure 3.3

The quadratic equation was no longer a dry formula or procedure. We enjoyed doing the problem and the class was greatly enlivened. The fascinating part for me was travelling back in time. I felt as if we were conversing with Bhāskarāchārya some 900 years later! There was also the realisation that there were great mathematicians in India – going back a thousand years and beyond –who had worked on what we were learning in school in 1984!

In the end, an otherwise dull exercise turned out to be exciting as we saw the practical value of the solution of the quadratic equation as well as its historical connection. It was the story that did the trick, like the story about Euclid and his approach to proofs, though Euclid's case goes further back in history and is a more complex one. We were beginning to encounter exciting topics in maths. "How old was the history of maths?" I wondered.

Before we think a little more about using the historical method to teach mathematics, here is another Bhāskarāchārya problem for you to munch on. This one yields a quadratic equation which needs to be factorised for a solution:

> "From a swarm of bees, a number equal to the square root of half the total number of bees flew to the lotus flowers. Soon after, 8/9 of the total swarm flew to the same lotus flowers. A male bee, enticed by the fragrance of the lotus, entered into a flower. But when it was inside, night fell and the lotus closed. The bee was trapped inside. "Oh my beloved," wailed his consort, responding anxiously to his buzz from the outside. How many bees were there in the swarm?"

The more I think about it, the more I'm inclined to believe that Channa was a history teacher and storyteller par excellence. Much better than our regular history teacher who killed history by reading from the textbook with his head bent, in a classroom that became noisier by the minute. Sometimes we just slept. It was a sheer waste of time. But it could also be fun, albeit unintentionally. Oblivious to our teacher, we would throw paper planes and rockets, chalk pieces and the like at each other during his boring rendition of the subject matter. I don't remember him ever looking up to admonish us.

How did Channa actually use history? To answer this question, we need to first ask another fundamental question: Why do students find mathematics difficult and uninteresting? There could be many responses to this question that might include poor teaching, inadequate preparation by the teacher and the like.

My response, based on my experience of being taught by Channa, is that we don't have teachers who explore the mysteries of mathematics enough. If this happens, they inevitably start asking questions: Why? Who? Where? How? They go into the history of the subject. They start looking for stories and patterns. They once again become students of the subject. That is critical.

But training teachers to explore mathematics is a challenging task. The challenge begins with the trainers themselves. Are they curious? Do they read, reflect and enquire? Are they even interested in what they are doing, in probing deeper into their subject? This is why we inevitably end up with what educationists call a 'systemic' problem in education.

The eternal challenge of education, as many a thoughtful parent or teacher has discovered, is to teach for meaning and understanding, to teach so that children understand 'why'. This challenge is even more acute when it comes to teaching mathematics. Can we learn to explain 'why', instead of succumbing to the easier option of ramming formulae and procedures down the throats of unsuspecting students? The latter is why mathematics does not seem to make sense to them and repels them.

Children are forever asking questions. They are eternally curious. We were all like this in our childhood. What happened to us when we grew up? Perhaps we need to rediscover the child within ourselves. This might help us respond to children in better ways.

How do we satiate their curiosity, which we no longer feel as adults? Perhaps, we need to tell them stories. Stories never fail to excite, stories of the people who created the mathematics we learn in school, stories of how mathematics developed over thousands of years. These stories must become an intimate part of mathematics teaching. Only then will children discover the true nature of mathematics and develop a fascination for the subject. Only when they experience the thrill of discovering and creating mathematics will they understand that this topic is something more than just a mindless jugglery of formulae and theorems.

In Channa's scheme of things, mathematics was not something that dropped down readymade from heaven, to be used by magicians to get great grades in the exams. He did not see the subject in isolation without the human angle and without and explanation of the 'why' of it.

In his journey as a teacher, Channa brought in the historical element to kindle our curiosity and make the subject more human. As we journeyed through high school with him, mathematically inclined or otherwise, he treated us to many fascinating stories. That was one of the key techniques of his teaching, perhaps one of the most effective ways in which one can fire the imagination of a learner.

Many of these hundreds of stories of mathematical discovery dating back several thousand years were rooted in everyday problems. They told us that mathematicians were not magicians but humans like you and me who led interesting lives. This was something that our rote learning and mathsphobia had hidden from us until Channa, with his natural interest in the subject, took us on a journey of discovery. He had the subject's history on his fingertips and knew what was happening in the mathematical world. We began asking more whys as we began to seek meaning and understanding instead of merely trying to pass our exams.

Channa's 'interdisciplinary' approach was in perfect unison with what Aristotle had said more than 2,000 years ago: "If you would understand anything, observe its beginning and its development."

Research into how children in schools learn mathematics shows us why history is important in its teaching. It can help in answering 'why' in several distinct ways. First of all, children have all kinds of 'why' questions about historical facts. We all did, if you can remember your schooldays. These include questions such as, "Why should a right angle have 90 degrees? Why are there 60 seconds to a minute and 60 minutes to an hour?" A well-informed teacher can throw light on how definitions developed in mathematics and why they are necessary, even though they maybe arbitrary most of the time.

Second, and perhaps most important, children ask 'why' about topics and concepts. For instance, why does one need algebra? Why calculus? Of what use are they? How were they developed? There is always a background, along with key players, to the development of new ideas in this field. Understanding this background helps us to establish the mathematical connections between topics. It also tells us what spurred the development of different ideas. Most of all, it humanises the subject matter.

The question of the utility of a mathematical topic or idea can be addressed in two ways. For the 'pure' mathematician, a new idea or a mathematical breakthrough furthers the cause of mathematics itself, irrespective of whether it contributes to human welfare or scientific development. For the 'applied' mathematician, the main concern is always about where and how maths can be put to use to enhance the material well-being of society. This concern is wide-ranging, encompassing all that happens in the sciences, engineering, social sciences, arts and commerce – in fact, in every area of human endeavour.

Whenever he took up a new topic, Channa would spend a few classes telling us a story related to that topic. He would explain how and why a particular idea developed over time and also talk about the key players who were involved in its evolution. He would simplify the narrative sufficiently for us to grasp the idea. Slowly but surely, as the story unfolded, he would make us understand how this idea was linked to what we were about to do (or were already doing) in the classroom.

Armed with this bird's-eye view of mathematical connections and perspectives, we would then get on to problem-solving, which was usually the more routine but equally important part. The stories remained with us for a long time. I remembered them even after so many years because they kept me thinking and imagining.

We could see that mathematics was not some isolated mumbo-jumbo. It emerged at different points in time in different cultures to serve different human purposes and expand the domain of human knowledge and experience. The purpose could be purely intellectual, or it could be to solve a specific problem in society or to aid the development of scientific thought.

Mathematics is thus intimately connected with human society. It came into existence to answer people's questions and solve problems of various kinds. It grew and evolved as more questions and problems emerged. It continues to invent new ideas to resolve the challenges facing human existence. If children can appreciate this, maths-learning can become that much more purposeful, and a sound foundation for learning the subject can be built.

I'm not saying that history is all there is to it. But it is indeed an important part that is often missing in the puzzle of teaching and learning this subject. The history thing can be a bit dicey, as we have seen with Euclid in the last chapter. It gets more complex once we bring in geopolitics. We will come to that later.

Channa comes across as very calm and self-assured both as a human being and teacher. He is aloof, almost Buddha-like, but radiates warmth. We cannot get close to him, or get very friendly either. He ensures that there is always some distance between us and him. Fair enough, I think. That's the way he is. But when it comes to learning maths with him, it's sheer fun. With that partly-bald head and bespectacled face and his immaculate suit, he looks quite the archetypal mathematician himself!

Every morning, Channa comes to school on his Lambretta scooter. What goes on in his mind and what does he plot for us every day?

When I look back on those three years that he taught us, I realise how grounded he was in his knowledge of the subject. All these years later, I realise that he was, indeed, an avid explorer of the mathematical world and its eternal mysteries and paradoxes.

4 Stubborn problems and paradoxes

"Perhaps the greatest paradox is that there are paradoxes in mathematics."
– Edward Kasner

Have you heard about the Seven Bridges of Königsberg, the Four-Colour Problem, the Barber's Paradox or Fermat's Last Theorem? If you haven't, let me tell you that these are among the most interesting and intricate problems in mathematics that have defied the best minds for generations, sometimes for hundreds of years! In their attempts to solve these problems, mathematicians invented entirely new branches of the subject. These, in turn, have taken mathematical investigation forward.

Channa told us these fascinating stories of enquiry down the ages. I remember them to this day. I have passed them on to my students in the best manner I could. I have often wondered why they were not included in our syllabus. It was almost as if the syllabus was designed to make our lives boring at school.

As our teacher, Channa wanted us to glimpse the process by which mathematics is actually created. So he shared with us some central problems that have contributed to the development of the subject. I recall four of these, which I would now like to place before you. Are you ready? Be assured that there is never going to be a dull moment.

First, the Seven Bridges of Königsberg – I must share this.

I'm not sure which grade it was – maybe the ninth or tenth, around the time we were about to learn the idea of matrices. Instead of plunging headlong into the topic, introducing matrices as, "an array of numbers… arranged horizontally and vertically," and then stating that, "these are the rules for their addition and multiplication," and following this up with boring problem-solving exercises from the textbook, Channa did something entirely different. He simply had to get to the root of the idea and share the excitement of his explorations with his students. And that is how we were treated to the 300-year-old 'Seven Bridges of Königsberg' puzzle.

I remember we spent a couple of periods discussing this puzzle, which originated in daily life. It informed the development of new areas of mathematics such as graph theory, which finds wide applications in science and engineering. Linked to this are matrices, which are used to simplify intricate visual representations in graph theory. They are also of great use in areas like quantum mechanics, a branch of physics that looks at the mysterious workings of the subatomic world. In fact, there is something called 'matrix mechanics'. Well, I'm not sure how the topic of matrices came into our school syllabus, but I'm glad it did, since we were treated to this wonderful story.

The Seven Bridges problem originated in the town of Königsberg, founded in 1254 CE in Prussia, now known as Kaliningrad in modern-day Russia. The town itself was made up of four land masses that were connected to each other and the mainland by a network of seven bridges built on the Pregel River, which ran through Königsberg. The seven bridges (not all of them exist now) were named Blacksmith, Connecting, Green, Merchant, Wooden, High and Honey.

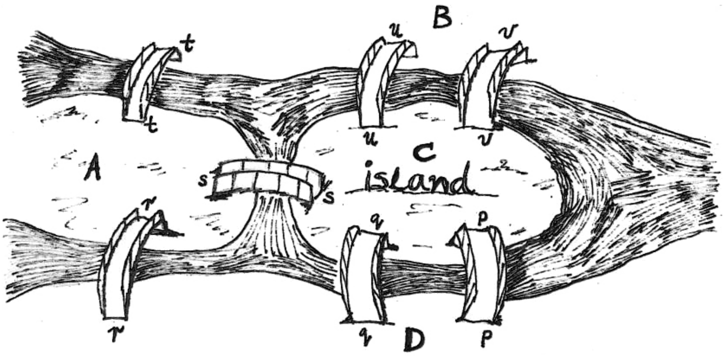


Figure 4.1

According to folklore, the challenge was to walk through the town of Königsberg in such a manner that one crosses each bridge only once to finish at the starting point. Why don't you try tracing such a path through the bridges in Figure 4.1? In 1736, at the invitation of the members of the St Petersburg Academy, Leonhard Euler showed that it was impossible to cross the seven bridges without retracing one's path. He also provided the criteria to solve any problem of this kind. Since the criteria do not apply to the Königsberg puzzle, it does not have a solution. Before we discuss it a little bit more, spare a few moments to see Euler's own sketch of the problem in Figure 4.2.

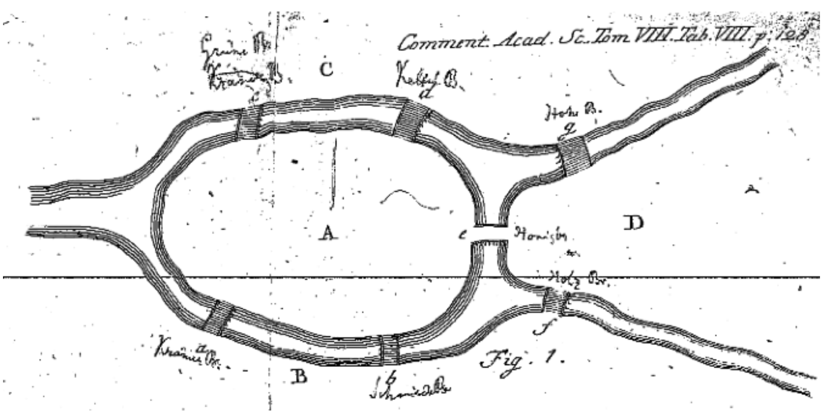


Figure 4.2

What Euler did was this: he reduced the network of bridges to a simpler-looking graph, as shown in Figure 4.3, with four vertices or nodes (A, B, C and D) and seven edges or paths (t, s, r, p, q, u and v).

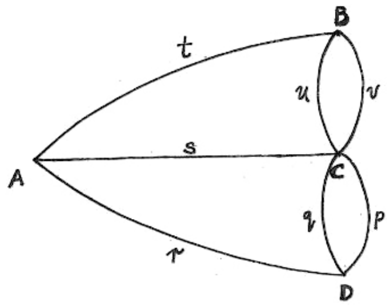


Figure 4.3

If you look carefully, this graph corresponds to the first illustration showing the bridges (Figure 4.1). In fact, it is a graphical representation of this real-world problem.

The question now is: Is it possible to find a path through the above graph such that each line is traversed (walked) only once, and you come back to the starting point – that is, can you do a closed tour? Try and see for yourself!

A graph of this kind, which is a visual representation of a phenomenon (in this case, a real-life phenomenon – the Königsberg bridges), can also be represented as a matrix, which is an array of numbers. As graphs get more and more complex, matrices become an effective tool to represent, manipulate and study them, especially when one uses computers for the task.

The Königsberg problem can be represented as an 'adjacency matrix', which looks like Figure 4.4. It contains information on how each of the vertices is connected to the others.

$$
\begin{array}{c c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix}
\end{array}
$$

Figure 4.4

You might wonder how we got the 0s, 1s and 2s in the matrix. On the graph, the number of paths from point A to itself is 0, the number of paths from point A to B is 1 and the number of paths from point B to C is 2. Likewise for all the other paths.

Sometimes, we use the term 'degree' to characterise the vertices we see in a graph. The degree can be odd or even. If you have an odd vertex, it means that the number of paths going out of that vertex is an odd number. In the Königsberg graph, A is odd and so are B, C and D. We say that their respective degrees are 3, 3, 5 and 3. When a graph has more than two odd vertices, it cannot be traversed without retracing a path. This is what Euler showed. This is why the Königsberg problem does not have a solution.

The matrix in Figure 4.4 shows the information contained in the graph in a simpler way. Add the columns A, B, C and D separately. What do you get? 3, 3, 5 and 3 respectively, right? That is how a graph can be replaced by a matrix. That is how people started using matrices to represent information from complex and intricate-looking graphs (the Königsberg graph is not so intricate, of course). In many ways, matrices can be used to represent real-world phenomena.

Matrices are also used to interpret the change in physical properties of subatomic particles over time. In high school, they are used to solve what are known as simultaneous equations, which contain two or more unknowns. In fact, matrices are so powerful that they can be used to solve simultaneous equations containing multiple unknowns – even as many as 20, 30… etc.

So that was it – what an introduction this was to the topic of matrices! For the first time, I could see how a certain topic arose in mathematics. It made me feel good. Even if we did not delve further, I was happy with the understanding that each of these mathematical ideas had a reason behind it. They did not just appear out of thin air to get imprinted in our textbooks. I suppose if you get to the root of the matter to answer the whys, learning becomes more purposeful.

We spent two classes on the Königsberg problem, trying to draw our myriad routes across the seven bridges only to be stumped in the end. Then Channa told us that Euler showed that the Königsberg problem could never be solved. With that familiar gleam in his eyes, he said that Euler gave his proof with the help of paper and pencil, without even bothering to walk across the bridges! Such is the power of mathematical thinking, he told us.

The development of this new branch of mathematics 'graph thory' and the related idea of matrices preceded the development of another branch of mathematics called ‘topology’, as Channa told us. For lack of time, he did not go into the details to enlighten us about these connections, nor did we have the time to actually see how Euler resolved the Königsberg problem. The proof is not so difficult to understand, as I discovered later, but during Euler’s time it must have been breaking news. Some mathematicians say the Königsberg problem is Euler’s ‘most famous work’ since it came to be so much in the public eye, though it was by no means his most profound achievement in the subject.

Our adventure of discovery continued with Channa, way beyond the dreary ICSE syllabus. My fascination for maths grew and grew.

At first, Channa said, Euler's ideas on graphs (arising out of the engagement with the Königsberg puzzle) were used only to solve puzzles and games in recreational mathematics. But the pursuit of knowledge can often throw up connections that one is not prepared for. It was gradually discovered that the idea of graphs could be applied to many other problems in the field, such as the Four-Colour Problem (FCP), sometimes also called the Four-Colour Theorem. That is how Channa took us to another famous enquiry in mathematics. More fascinating discussions of intellectual struggle and discovery ensued. How I wished we didn't have exams!

The FCP was solved by the mathematicians Kenneth Appel and Wolfgang Haken just two years before I joined Baldwins in 1978. Originating in map-making and cartography, it remained a great unsolved problem for 124 years from 1852 to 1976. It all began with an innocent question that a chap called Francis Guthrie, who was a student at University College in London, asked his brother sometime in 1852. Out of curiosity, Guthrie asked if every map could be coloured in such a way that no two regions sharing a border have the same colour. Apparently, this question occurred to him when he was colouring a map of England. I'm not sure why he was colouring the map but sometimes the most innocuous of activities can lead us to profound insights about the way things work. Anyway, little did Francis realise that his question would unleash a storm in the world of mathematics.

The problem was eventually solved in a way that the Guthrie brothers could never have imagined. The FCP has no simple paper-pencil proof that can be verified by just about anybody. For the first time ever, supercomputers had to be used to process the huge amounts of data required to 'prove' a major theorem in mathematics. This voluminous data was generated because there were attempts to look at smaller configurations on the map with 'n' regions and then see whether the assumption of four colours being sufficient for the task held true. On any map with ‘n’ regions, one can have a very large number of configurations.

There was a debate within the mathematics community whether a computer-aided method constituted mathematical proof. Mathematics uses deductive proof based on logic, Euclidean style, whereas this seemed more like an experimental proof that the natural sciences routinely use.

Appel and Haken broke the mould by using computers to prove mathematical theorems for the first time ever.

To put things simply, the statement of FCP is like saying that one does not need more than four colours to colour a map such that adjacent countries or regions do not have the same colour. I suppose this is easy to understand and sounds deceptively simple.

A more rigorous statement might look like this:

> *“Given any separation of a plane into contiguous regions, producing a figure called a map, no more than four colours are required to colour the regions of the map so that no two adjacent regions have the same colour.”*

A simple map is shown in Figure 4.5, needing four colours to colour the map in such a way that no two adjacent figures have the same colour. This looks unlike the maps one is normally used to, since it is made from geometrical figures having straight lines. It is a map nonetheless.

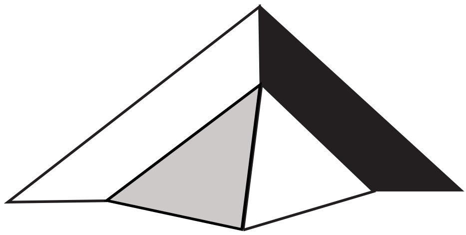


Figure 4.5

While mathematicians were able to prove the case with five colours, the FCP stubbornly resisted a solution for well over a century. Interesting, isn't it? One can never be sure which area of human activity can actually spawn a new area of knowledge that people keep pursuing even hundreds of years later! So, after all that drama, we now know that four colours are enough, whatever be the complexity of the map, and both on a plane and a curved surface.

Try this. Get an uncoloured political map of India (the one that has the border of each state clearly shown) – the ones that kids usually do their homework on. Start painting each state with a different colour. See if you can do it with three, four or five shades, as long as your colouring satisfies the condition that no adjacent states (that is, states bordering each other) have the same colour. Got a feel for the FCP? Innocuous though it sounds or looks, it couldn't be polished off for more than a century.

And so, Channa took us all on a great mathematical adventure. He would always enliven the class with stories, reminding us that there was more to maths-learning than what was given in our syllabus. Perhaps the more interesting parts were outside of it.

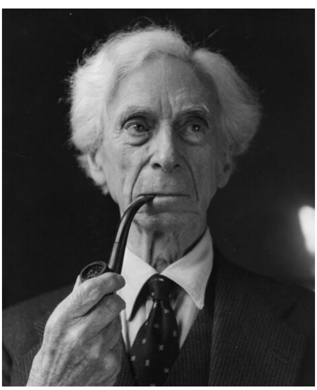
The months went by. We meandered into 'set theory'. And Channa's stories only got more intriguing. At times, they even sounded funny. Before we got into the routine of learning the standard definitions in sets, Channa suddenly announced one day, *"Let me tell you the story about this barber in a village."*

Barber? We looked at him in anticipation. He went on, "Suppose there is a certain village which has a barber who shaves all and only those inhabitants who do not shave themselves. Does the barber shave himself, or doesn't he?"

Let us think this through a little bit. The barber does not shave any inhabitant who shaves himself. He shaves only those inhabitants who do not shave themselves. You land up in a sticky situation when you try to answer the question: Who shaves the barber?

If he shaves himself, he violates the rule that he never shaves anyone who shaves himself. If, on the other hand, he doesn't shave himself, then he is one of the inhabitants who don't shave themselves. Then according to the rule, he must shave himself.

Voila! We have a contradiction here. It takes a while to sink in. The problem looks simple at first sight but when you grapple with it, you get tied into knots. When it was discussed in class, I remember visualising an unshaven barber whose beard kept growing and growing indefinitely. "To shave or not to shave?" the poor barber must have asked himself a million times.



Russell (1872-1970 CE) set the cat among the pigeons with the barber's paradox, also known as Russell's paradox. It was to have far-reaching implications for the rest of mathematics.

Anyway, we had fun confronting this barber's dilemma, which Channa said was known as the 'Barber's Paradox'. It was first proposed by the mathematician and philosopher Bertrand Russell near the start of the twentieth century. I remember the loud arguments and counter-arguments in class on the paradox. We couldn't quite get around it.

That is how we were introduced, for the first time, to the term ‘paradox’. None of us knew what this creature was. Channa explained that a paradox in mathematics occurs when we encounter a statement that contains ideas and thoughts which seem to be true but are conflicting. For example, take a look at the two following sentences:

> *“This sentence does not have seven words.”(Whereas it actually has seven words!)*
>
> *“This sentence has seven words.”(Whereas it actually has only five words!)*

This is a simple way of understanding what a paradox means. These sentences contradict themselves! How can they even exist? But there they are, staring at us. That’s what a paradox is.

The Barber’s Paradox, of which there are many variations, is also called a 'paradox of self-reference’. Many maths paradoxes fall into this category. Channa pointed out that the Barber’s Paradox exposed a contradiction at the heart of the branch of mathematics called ‘set theory’. He wanted us to understand that the topic of sets has witnessed some of the most intense contradictions and battles in the development of mathematics.

In simple terms, a paradox means that there is a statement ‘S’ such that ‘S’ and its negation (‘not S’) are both true (as seen in the case of the barber). Such inconsistencies can make the foundations of the entire subject very shaky, since we would then have no basis for trusting any mathematical proof. Remember when we discussed the angles of a triangle theorem and its proof earlier, Channa insisted that the proof must be solid and robust, no matter what kind of triangle one considered?

To illustrate paradoxes like the self-referential Barber’s Paradox, reflect on one of the versions of what is famously known as the ‘Liar Paradox’ (which I remember was also discussed in Channa’s class): “All Cretans are liars.” This ancient paradox is usually attributed to Epimenides, a Cretan, who is said to have lived in the sixth or seventh century BCE.

Or consider the following statement: “This statement is false.” Is the statement true? For the statement to be true, it actually has to be false. And if the statement is indeed false, it is true. Something crazy is happening here!

All the above statements result in contradictions, like in the Barber’s Paradox. Think about them! Get consumed. These paradoxes grow on you. Whoever imagined that mathematics had all these crazy twists and turns? Channa would survey the classroom after each heated discussion that we had on these paradoxes, usually with a smile of satisfaction that only a teacher who has aroused agitation in the learner to discover more is entitled to.

As I learnt later, the brilliant Austrian-American logician and philosopher Kurt Gödel showed from the paradoxes of self-reference that mathematics is and will always be ‘incomplete’. If mathematics is indeed incomplete, then what about science whose backbone is formed by this subject? Can we then say that the knowledge we glean from science about everything will always be incomplete? These are intriguing questions. I have tried to elaborate on them a little more in the additional notes section given near the end of the book.

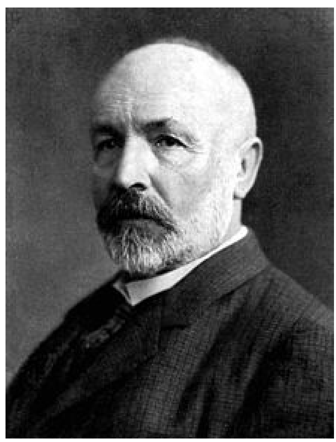
We then moved on to a brief discussion about infinity. This was, again, in the context of sets. I remember Channa saying rather ominously, “Infinity is not a number because if it is one, you can always add one more and think about the next number. This can go on endlessly and we will end up getting well-defined numbers. Therefore, we cannot treat infinity as a number and use the usual operations of arithmetic to deal with it.”

This stumped me quite a bit. Then what was infinity about, if it was not a number? How could we deal with it?

As we discussed infinity, there was this argument about the number of sand particles on a beach. Was their number finite or infinite? I think the argument came up in our discussions about what could or could not constitute infinity. I was among those who argued that the number of sand particles on a beach is infinite while there were others who said that it is finite. My logic was that each sand particle could, in principle, be infinitely divided. How could it then be said that the number of particles is finite? What do you think?

What was intriguing about infinity was that it could not be treated like any other number. It was also interesting when Channa said that there are 'different orders of infinity'. For instance, which set of numbers is bigger – the set of all even numbers or the set of all natural numbers? Both sets of numbers are infinite, but can we say that one is bigger or more infinite than the other? Is there a way of comparing infinite sets?

Much later, I learnt that the 'real numbers' (which includes integers, fractions or rational numbers, and irrational numbers) between any two points on the number line are more numerous than the set of 'natural numbers' (which are the numbers we use to count, like 1, 2, 3, etc). This is puzzling, isn't it? Using a proof that mathematicians often describe as the most elegant proof ever, the German mathematician and founder of set theory, Georg Cantor, showed that real numbers belong to a higher order of infinity than natural numbers.



The path-breaking work of Georg Cantor (1845-1918 CE) enabled mathematicians to deal with infinity in more rigorous and precise terms than ever before.

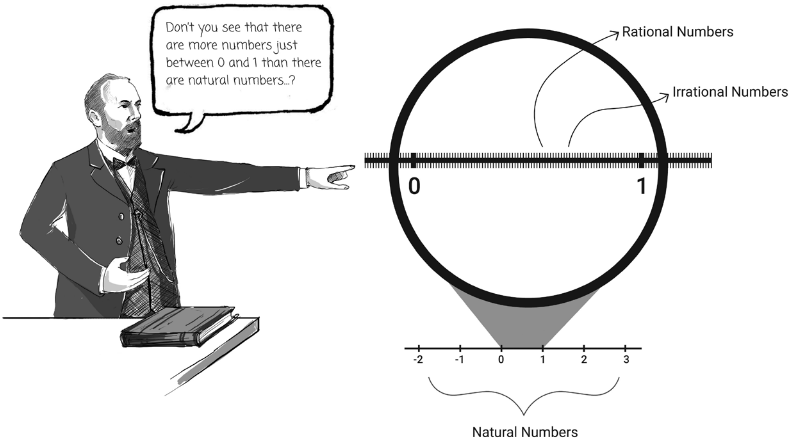


Figure 4.6

Cantor introduced the idea that infinity could literally come in different sizes. In fact, he asserted that there is an 'infinity of infinities'! He faced intense, often derisive, opposition from his fellow mathematicians for proposing such ideas. Many of them recoiled in horror at the thought of engaging with the idea of the infinite, the most well-known among them being Henri Poincare, who referred to Cantor's work as a 'grave disease' afflicting mathematics. But Cantor's ideas provided his contemporary mathematicians with a tool to tackle infinity.

And finally, the big one. Who can forget 'Fermat's Last Theorem'? We were discussing the proof for the Pythagoras Theorem in Channa's class: That the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides. So $a^2 + b^2 = c^2$, where 'a' and 'b' are the two sides of the right-angled triangle and 'c' is the hypotenuse. (Remember our discussion of Bhāskarāchārya's problem in *Lilavati* earlier, which uses the Pythagoras Theorem to get to the solution?)

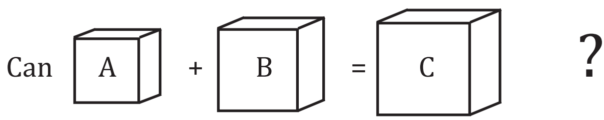
Though it is difficult to ascertain, at last count, there were as many as 367 distinct proofs of the Pythagoras Theorem! I use the word 'distinct' because in order to be counted, each proof has to be different from all the other proofs. A book called The Pythagorean Proposition by an early twentieth century professor Elisha Scott Loomis presents a collection of these 367 proofs. What is amazing is the sheer variety of approaches that exist to establish the same truth. On this list is also the stunning visual proof of Bhāskara. He was supposed to have famously said "Behold!" while showing his illustration of the proof, which did not need any words in its description.

Every high school kid knows the Pythagoras Theorem but it is quite unlikely that many have gone beyond the theorem itself. Well, Channa did just this. We were treated to Fermat's Last Theorem (FLT), which had remained unresolved for 348 years. Just what is FLT all about? Well, good old Pierre de Fermat (pronounced 'Ferma', the 't' being silent), a French lawyer by profession who doubled up as a maths enthusiast, was looking for 'Pythagorean triples' around the year 1650 CE. So goes the story.

Let me share some examples of sets of numbers that are given the name of Pythagorean triples. {3, 4, 5}, {5, 12, 13}, and so on are Pythagorean triples since $3^2 + 4^2 = 5^2$, $12^2 + 5^2 = 13^2$. There are various ways in which we can keep generating these triples. For instance, we can multiply the triple by a common number or factor like 2. Then {3, 4, 5}, when multiplied by 2, becomes {6, 8, 10} and it is easy to see that $6^2 + 8^2 = 10^2$. What this means in geometric terms is that we can construct a right-angled triangle whose sides are 6, 8 and 10 units each, where 10 units is the length of the hypotenuse. One can now multiply this triple by a factor of 2 and get another Pythagorean triple. This can go on endlessly. There are other ways as well of generating such triples.

Slowly, Channa took us closer to the twist in the tale. He said that Fermat asked a rather simple question, which must have sounded like: "Is it possible to do it with cubes – that is, can we find the three positive numbers 'a', 'b' and 'c' such that $a^3 + b^3 = c^3$?" He was only looking for a pattern. Remember, a good indicator of maths-learning is that the student has developed the interest and ability to look for underlying relationships and patterns. This is the test of character for a mathematician too.

That things don't work when n = 3 goes against our intuition. One would have thought that if the two squares on the two sides of a right-angled triangle can fit into the larger square on its hypotenuse, then the same logic should apply to solid cubes as well. Right? Well, when a cube comes into the picture, we are talking about third powers (like a^3). A cube is a three-dimensional object bounded by six square faces that are placed at right angles to each other.



(Can $A^3+B^3=C^3$?)

Figure 4.7

In Figure 4.7, A^3 can be thought of as a cube with a side of A units. Similarly for B^3 and C^3. The volume of a cube (the space it occupies) with side A is (A x A x A) or A^3. Likewise for cubes with sides B and C respectively. So when we try to prove that there are three positive integers A, B and C such that $A^3 + B^3 = C^3$, we can visualise this problem geometrically as one of exactly fitting cubes A and B into the bigger cube C, if it is hollow. But mathematics can often go against common sense, such is its nature. It turns out that $A^3 + B^3 = C^3$ does not hold true!

Well, there is one set of numbers that gets painfully close to proving that $A^3 + B^3 = C^3$ is true – {12, 10, 9}. Consider $(10^3 + 9^3)$, which works out to 1729 – the famous 'Ramanujan number'. And 12^3 happens to be 1728 – phew, pretty close. Something like a very, very close 'leg before wicket' decision in cricket!

We can visualise the problem geometrically when we deal with n = 2 (in this case, it is the Pythagoras Theorem) and n = 3. But how do we visualise it when n = 4, 5, 6…? We can no longer take refuge in the two- or three-dimensional objects that we are accustomed to dealing with in our daily lives to help us out. Things become abstract at this stage and you need other ways of looking at the problem.


Pierre de Fermat (1601-65 CE) – lawyer and 'prince of amateurs' – was the greatest mathematician of the seventeenth century. His challenge to posterity was finally solved in 1994.

But mathematicians cannot rest till they come out with solid proofs that Fermat's theorem does not hold for all 'n', where 'n' is a positive integer and n>2. They are not restricted by the limitations of the physical world and routinely deal with 'n' dimensions in their work.

Channa told us that Fermat kept on asking: Can we do it with the fourth powers – that is, can we find the numbers 'a', 'b' and 'c' such that $a^4 + b^4 = c^4$? He eventually asked: What about three numbers 'a', 'b' and 'c' such that $a^n + b^n = c^n$? Fermat must have tried to obtain the proof for the various powers of 'a', 'b' and 'c'. It is said that he did prove it negatively for the fourth power (n=4), meaning he proved that the equation $a^4 + b^4 = c^4$ does not exist. We do not know whether he similarly proved that equations containing higher powers of 'a', 'b' and 'c' (for any positive value of 'n') do not exist. However, he finally proclaimed in Latin:

> *"Cubum autem in duos cubos, aut quadrato-quadratum in duos quadrato-quadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet."*

Translated into English, this is what it reads as:

> *"It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers or, in general, any power higher than the second, into two like powers. I have discovered a truly marvellous proof of this, which this margin is too narrow to contain."*

In other words, no three positive integers 'a', 'b' and 'c' can satisfy the equation $a^n + b^n = c^n$ for any integer value of 'n' greater than two.

The statement: "I have discovered a truly marvellous proof of this, which this margin is too narrow to contain." was scribbled in the margin of Fermat's copy of a book titled *Arithmetica*, written by that great Greek mathematician Diophantus who lived in the third century. This copy with Fermat's comment was discovered 30 years after he died in 1665. One does not really know if the lawyer had figured out the proof. Many believe he did not, simply because the tools to prove FLT were not available then. Perhaps no one will ever know.

But Fermat's statement gave birth to one of the most notorious problems in mathematics that remained unsolved for nearly 350 years, sending mathematicians across the globe into a tizzy. It caused so much fuss that it even attracted a huge sum as prize money when the German industrialist Paul Wolfskehl donated 1,00,000 marks in 1908 as a reward for anyone who could prove FLT!

When Channa finally completed this story, he said rather sombrely, *"Maybe one of you will solve it one day!"* We could sense his disappointment.

Before I move on, let me leave you with two sets of three numbers that may just disprove Fermat and his last theorem. Check them out, try your luck, stake your claim to fame and share the spoils with me. Here they are:

The first set is: $3987^{12} + 4365^{12} = 4472^{12}$

The second set is: $1782^{12} + 1841^{12} = 1922^{12}$

The mid-1980s were exciting FLT times. Countless mathematicians had by then grappled with the theorem and proofs had been painstakingly built up and presented for a wide range of powers of ‘n’. By Baldwin’s centenary year in 1980, FLT had been proved for all powers of ‘n’ up to 125000! But this number is miniscule when compared to the infinity of positive integers. What we need is a general proof applicable in every single case. For how long can we go on proving specific cases? Remember the proof of the sum of the three angles of a triangle? We had struggled so much when we encountered the notion of proof for the first time. It was so different from solving any other problem. The Monstrous Counter Example was another such exercise.

Little did we realise when we were in grade 10 in 1985 that Andrew Wiles, a British mathematician, was close to deciding that he would spend the next seven or eight years of his life trying to crack FLT in total secrecy. By then, new branches of mathematics had been invented. Those came to Wiles’s aid, his rather roundabout proof completely filling all of 150 pages.

I wonder what Channa must have felt when the problem was finally declared as solved in 1994. What did he discuss with his students then? Wiles’s story is fascinating but if I talk about it here, we will move away from Channa. There are several popular books that tell the story of the FLT struggle and explain its proof. You can check them out.

FLT is a great example of the triumph of the human mind and spirit against all odds. Wiles’s story shows what the indomitable will of an individual can achieve. It is as gripping as any other story of human struggle. It shows that if we persevere in pursuing our dreams, we can get there one day, even against enormous odds.

Mathematics can be the beginning of such a journey for every student in school. The task of education is to create the conditions for every student to be able to access this beautiful subject. It is no easy task. We are up against an entrenched culture of rote learning, a culture which does not engage in an enabling way with children from diverse and often challenged backgrounds.

Channa kept taking us on these fascinating detours time and again. I must mention my consternation when we encountered the number pi (π), which belongs to a class of numbers called 'irrational numbers'. Now, these are numbers that simply do not behave 'rationally', with their fractional parts going on and on and on in a never-ending stream of decimal digits. It still astounds me that such numbers can even exist. We learnt from Channa that efforts to pin down π had reached gargantuan proportions by that time. Using the fastest supercomputers then available, it was discovered that the elusive π has an unending and non-recurring stream of decimal digits even after 16 million places! I was perplexed. They have computed π to several trillion places by now, yet the decimal digits keep going on and on.

It still gives me goosebumps when I recall those few classes in which we got a deeper glimpse of mathematical reality, far removed from the mundane world of passing tests and winning the rat race, far removed from 'completing the syllabus'.

Channa had a simple objective – to get us to appreciate that mathematics is not a dead subject but an evolving human story that holds a lot of interest and thrill for those who are inclined to its study. Even more important, the idea was to keep those of us who were less inclined or even put off by the subject altogether, engaged on this journey of exploration.

Why don't they put all these things in the syllabus? They leave the best parts out! I know you are tired of hearing me say this, but I feel that it is worth pointing this out again and again… and again.

March 15, 1985. Our tenth standard exams were over, much to my relief. It was Graduation Day! They called it 'Commencement Exercises' in Baldwins, as if something new was about to commence in our lives.

We assembled in Lincoln Hall as sunset approached. After the usual lectures about leaving school and this and that, I remember lighting a candle and walking down the aisle, out of Lincoln Hall. Out, carrying the spirit of Baldwins wherever I would go in life. That is how I suppose it was meant to be. All of us wore suits, looking like young men, as Channa said. Anna had one of his coats altered to fit me, for the occasion. It was the first time ever that I had worn a suit.

We had an early dinner. School's treat! For the first time, I had a full meal at school. And then the goodbyes, handshakes, hugs. Teachers and friends not knowing when we would see each other again. It all happened in a flash.

Quietly, I made my way to the main gate, sweaty and stifled in that suit of mine. Traffic was heavy on Hosur Road. It was almost dark. A friend from Section A called out. He wanted a photograph of the two of us. We stood in front of the gate, and someone clicked his camera. I wonder if that photo still exists. That's when it hit me – my last day at Baldwins!

I cycled home in a daze, my suit stuffed in my bag. It would take some getting used to, not having to go to Baldwins any longer.

Aaji was alone at home, anna was on a late evening shift and amma was battling high sugar levels in the intensive care unit of a nearby hospital. I went first to the hospital, grateful to god that her diabetes was finally under control.

Gingerly, I stepped out into another world, carrying my myriad memories of school. Not knowing the ways in which they would influence me years later.

5 We teach as we were taught

"The mediocre teacher tells. The good teacher explains. The superior teacher demonstrates. The great teacher inspires."
– William Arthur Ward

September, 1993. I am sitting across the desk from the principal in his room at The Valley School. I want to become a teacher and I want to spend my time with children. Something tells me that my existence will then become more meaningful. I want to move on from my boring two-year-old job of manufacturing tractors for wealthy farmers whom I've never met.

Yes, I did take up a mechanical engineering course after my twelfth, but that was more out of peer pressure and family expectations than anything else. Now I want things to change. I am angry with our insipid educational system that makes us learn mechanically, that robs the life out of learning. I feel a surge of idealism and I want to change the world by becoming a teacher.

I believed then, as I still do now, that education is really the starting point to make another world possible. So I checked out some 20 schools before landing up at this one. Those 20 asked me for a BEd or MEd Degree. From where was I supposed to get them? At that time I was not convinced that I needed them. I was just exploring the idea of teaching and had this instinct that I would be a good teacher.

In the tractor-manufacturing company where I worked, my boss passed his judgment and told me that I was an escapist, that I was running away from life. “You are too young to realise what you are doing,” he nodded gravely when I walked into his room one day to try and explain why I wanted to move on to something else. He just wrote me off. At that time, I didn’t have an answer, a retort of sorts to counter his taunt about escaping from life. Was I really an escapist? The question kept coming back to me.

Some months later, after I had become a schoolteacher, I went back to him with more confidence and told him on his face, “I’m running into life, and I’m happier for it.” He still didn’t understand why I had become a teacher in a school. I couldn’t care less. I had taken my decision and was excited about this new journey. After all, I was following my heart.

The principal across whose table I am sitting says gently but tentatively, “We are looking for a math teacher. We’ll see you for a couple of months and then decide… the children will need to like you first.” I was relieved that no one asked me for a BEd or MEd certificate.

Two months later, they didn’t have a problem giving me the job. I guess I was a decent teacher. That’s the image I have of my own teaching.

Luckily, no one breathed down my neck to monitor my sessions in the classroom. I was not asked to submit lesson plans every week, as teachers need to do in many private schools. The school management trusted us. This was a progressive school, inspired by the ideals of the philosopher and teacher Jiddu Krishnamurti. So we teachers had greater leeway to experiment.

I mainly taught maths and physics in the middle and high school classes. For a while, I also taught the primary classes. It was an entirely new experience. I learnt a lot. There was this sense of being alive every moment. It was refreshing. Teaching, I realised, was a great way of educating oneself. We asked interesting and fundamental questions about ourselves and about the world in which we lived. These questions enabled one to lead a fuller life.

Inspired by Channa, I tried to employ the range of techniques that he used as a teacher – teaching fundamental principles, getting students to think, for instance. And then, prodding and nudging them, but never spoon-feeding them. And telling them stories from the fascinating and intriguing history of mathematics. I made it a point to discuss this history in my classes. The Valley School library offered me good resources and I read a lot. There were interesting conversations with fellow teachers as well.

Whenever possible, I took my students on a historical trajectory. We solved ancient problems based on the Pythagoras Theorem, in trigonometry, analytical geometry, logarithms, and widely debated paradoxes in mathematics. I included many of these discussions in worksheets that I made and even managed to give problems based on them in the test papers that I set. Much of what I did with the blackboard as a teaching aid was what I had learnt by observing Channa. I tried using the blackboard space well and as neatly as I could.

I even dabbled in 'Vedic mathematics', demonstrated some of its shortcut methods, got children excited about it and got involved in debates with my colleagues about its authenticity as a mathematical system. Did it actually originate in the Vedas, as some persons claimed? I remember attending a workshop on Vedic mathematics organised by the local RSS shakha in Chamarajapet in Bengaluru. We were trying to solve cubic equations, and the 'magical shortcuts' were on full view, eliciting frequent gasps of admiration from the audience. But when I asked the resource persons to change the coefficients of 'x' in the cubic equation they had written up on the board, the Vedic methods faltered. This raised questions about the claims that the mathematics was from the Vedas. Anyway, my students lapped up whatever I taught, as long as the methods worked.

This discussion about Vedic mathematics illustrates a larger question that I must now address at the risk of a brief digression. It is the question of the pursuit of truth and its implications for education. I had alluded to it briefly in the chapter on proofs, where I stated that a mathematical proof is an attempt to establish the truth of a claim within a particular frame of reference.

As we have seen, the study of mathematics helps us arrive at an understanding of mathematical truths, such as theorems for instance. But mathematics is also a great tool that helps us unravel the truths of nature. Why this should be, we are not sure. The power of mathematical logic and reasoning also makes mathematics an indispensable tool in understanding how a society works. I have stated all of this earlier, so let me move forward now.

I would like to echo the American philosopher and educational thinker Neil Postman, who once famously said that the task of education should be that of 'crap detection'. In my opinion, learning mathematics can help us immensely in crap detection and move us towards establishing a truly democratic society. This is the social function of mathematics, if one may call it that.

What is this 'crap' that one needs to detect? Often those in positions of authority use religious, cultural, political and identity-based arguments to propagate their fondest beliefs including misconceptions, faulty assumptions, superstitions and even lies as truths in order to retain power and control over others. Curricula and textbooks are easy means to spread these 'truths' throughout society. The history of our education is replete with many such examples. In today's age, social media has become a potent weapon for communication. Increasingly, we are being asked to take recourse to emotions and beliefs even when facts fly in the face of these beliefs. This is the 'crap' I'm talking about which education, especially mathematics education, must expose. Why take it lying down?

Voices of critical enquiry and dissent are often dubbed as sedition in the surcharged atmosphere of our times. I'm likely to be lampooned if I publicly question Vedic mathematics or other dubious claims about our identity, past and culture. These claims are all marshalled to restore the greatness of our heritage. So I argue that a culture of critical enquiry, of which the learning of mathematics is an integral part, can help make children experts in detecting 'crap'.

Let me get back to my experiences as a teacher. My students and I experienced several 'Aha!' moments in The Valley School. I still remember how awestruck a group of grade 9 students were when I showed them with the help of a small table (Figure 5.1), how the idea of a logarithm actually works - by simplifying multiplication and division operations into easier addition and subtraction. This is the idea central to logarithm that we rarely communicate to the student.

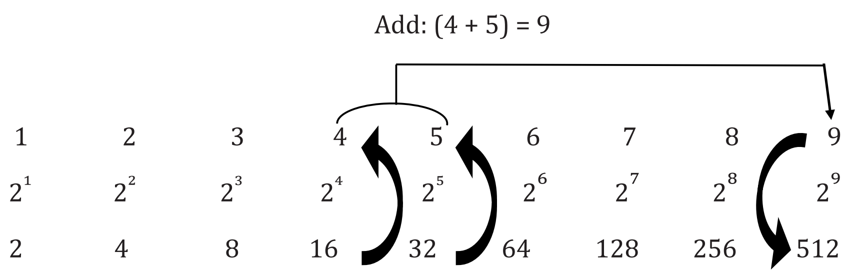


Figure 5.1

The numbers in the first row belong to what is called an 'arithmetic series' (where we just list out the natural numbers) and the numbers in the bottom row belong to a 'geometric series' (where each number is obtained by doubling the previous number). The numbers in the middle row, as you can see, express the numbers in the bottom row using 2 as the base. Thus, 16 is written as 2^4 and 32 is written as 2^5. Here, 2 is referred to as the base, while 4 and 5 are the exponents. We can use other bases as well but, for the sake of our discussion, let us keep to this example.

Suppose you need to multiply 16 by 32 (16x32). You look up the numbers corresponding to 16 and 32 in the first row – that is, 4 and 5 respectively. Add these: 4+5 = 9. What is the corresponding number for 9 in the bottom row? It is 512. That is your answer! Isn't this amazing? Once the class 9 students got the hang of this, they said, almost disbelievingly, "This is it?!" "Yes," I replied, "this is how the concept of logarithms works." In this case, the multiplication is actually performed through an addition, which makes it simpler and faster as well. A similar logic holds for division, which is performed through subtraction. Can you work that out for the division 256/8 with the help of the above table?

Since this is not a class on logarithms, I will not discuss the topic any further. I'm sure you have many questions. For instance, how to multiply 20x17? Or better still, how to multiply 3.2x5.4? And so on. What happens when things get more complicated? You can think of all kinds of division problems as well. And instead of 2 you can have any other number as the base. The logic I have presented will remain the same. A few more things need to be said, though.

Remember those logarithm tables with all their finely printed numbers that we would refer to when solving such problems? They are all constructed on the logic of our table in Figure 5.1. The first row consists of the logarithms, which basically means the power to which the base 'a' has to be raised to get a specified number. In our multiplication example of 16x32, let us take the number 16. What is the power to which 2 (the base) has to be raised in order to get 16? The answer is 4, since $2^4 = 16$. Similarly, for the number 32, the power is 5, since $2^5 = 32$. We usually write this as: $\log_2 16 = 4$ and $\log_2 32 = 5$.

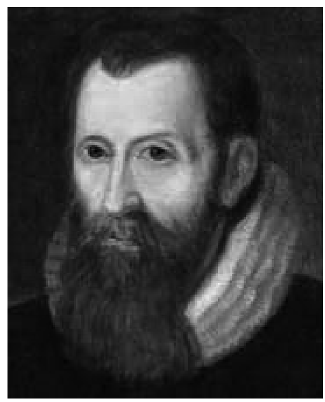
Here, 4 and 5 (the exponents) are called the logarithms. So when we write 'log' in front of a number, what we mean is: find out the exponent. We then use a simple rule:

$a^m \times a^n = a^{m+n}$

In this example, $2^4 \times 2^5 = 2^{4+5} = 2^9$

So, logarithms help us to add the logs: $\log_2 16 + \log_2 32 = 4+5 = 9$, thereby simplifying the multiplication to a simple addition. But 9 is obviously not the answer. So we now have to refer back to our table to see which number in the bottom row, consisting of the so-called 'antilogarithms', corresponds to 9 in the logarithms row. The answer is 512, as you can see from our table.

The fundamental principle of logarithms (and the logarithmic table) was known to the great Greek mathematician Archimedes way back in the third century BCE. But it was left to the Scotsman John Napier to provide the details around the seventeenth century, more than a millennium later. There was a practical necessity for these details, given that trade and navigation had expanded across the globe, which required routine calculations involving tedious numbers. Logarithms saved the day. Like Fermat, Napier also pursued mathematics as a hobby.



Scottish mathematician John Napier (1550-1617 CE)

Knowledge is painstakingly built. Hence, efforts to get children to understand knowledge need to be carefully thought out. But once they begin to find meaning in this painstaking task, there is no stopping them.

I told my students the story of logarithms in a couple of sessions. Imagine if I had begun instead by saying: "If $a^x = y$, then 'x' is called the logarithm of 'y' to base 'a'..." Wouldn't you have pulled your hair in despair? This is why children run away from mathematics.

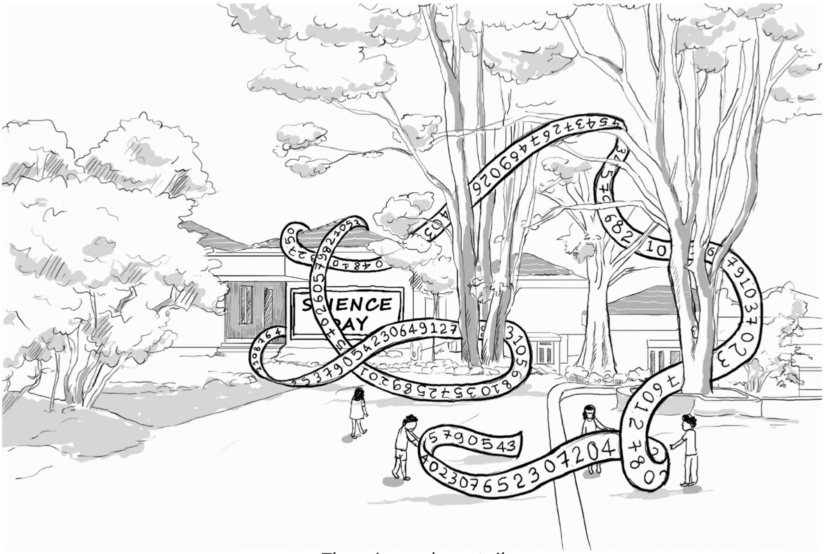
I don't remember Channa discussing the irrational π much with us beyond stating briefly that it is a non-terminating decimal. But as a teacher, I devoted quite some time to unravel this crazy number, which incidentally has a day named after it – π day – which falls on March 14 every year (can you guess why?). I must share with you an activity that we did to understand π. My students loved it.

As part of our school Science Day celebrations, we prepared a 'π tail'. I had with me a computer-generated value of π up to 2,500 decimal places that I had photocopied from a book in the library. The idea was to write out all these decimal places in the form of a tail and 'wrap up' the school with this tail. So we made 4-inch strips from newspapers, joined them and wrote out these 2,500 numerals using marker pens! Students would come to the math room whenever they were free to write the decimal digits on the tail. They often bunked classes to complete this irrational task.

On Science Day, we took out this long tail (which measured all of 850 feet) and starting from the noticeboard opposite the library, we literally tied the school up in the irrational grasp of this capricious number!

Everyone was curious, particularly the younger children. They were seen running along the length of this irrational tail. Up and down they went, amused at the unending numbers meandering around the school building, inside classrooms, coming out through the windows, even entering the toilet for a while before finally going upwards to get entangled in the jungle gym tree. When a few of my students came up to me and said, "Now we understand why one calls π an irrational number... it just seems to go on and on!" I knew the irrational π tail had made a difference. So much for 'experiential learning'. I'm not sure though, if they thought about π after that.

The other chance I got to take a jab at π was in an English literature class. Now, you can use mnemonics to remember as many decimal places of π as you want to. However, this is just a small consolation if you want to come anywhere near conquering this mysterious number.



The science day π tail

Figure 5.2

I was once invited as a math teacher to a creative writing class in English. I saw the invitation as an opportunity to 'integrate' disciplines in school. I wrote out the first 100 decimal places for π and asked the children to write a paragraph in such a manner that each word they used would have as many letters as the value of that digit, in that sequence. For example, the first seven digits of π (3.1415926) could be written thus: "May (3) I (1) have (4) a (1) large (5) container (9) of (2) coffee (6)?" Following this rule, the students wrote some hilarious stuff that made us all roll with laughter for the next few days. Unfortunately, I did not collect their samples.

As you can see, Channa's legacy lived on strongly in my life when I became a teacher. That's what a good maths teacher can do to you.

In 1994, an important event took place in The Valley School, which quite radically altered my understanding of the history of the development of mathematics. I learnt something more and took off from where Channa had left us. One of my senior colleagues, who was also interested in finding out how mathematics was developed across different cultures over the millennia, came to know that a maths scholar from Manchester University, George Gheverghese Joseph, was in town. Joseph had extensively researched the 'non-European roots' of mathematics.

We went and met him, to invite him to deliver a lecture at the school. To our delight, he readily agreed. Joseph's lecture was gripping and covered a vast canvas. He told us that his research showed how 'Eurocentric' the entire enterprise of mathematics and science has been. Indeed, to this day, we are taught in schools that Europe was the centre of global mathematical and scientific development since the days of antiquity. Such a blinkered view ignores the fact that other ancient cultures also contributed a lot to mathematics, often predating the discoveries made in Europe by hundreds of years, especially during the Dark Ages, that continent's great slumber.

Joseph talked about his pet research project in the context of this new understanding – the discovery of the 'Kerala school of mathematics' that flourished between the fourteenth and sixteenth centuries, most notably through the work of mathematicians such as Madhava of Sangamagrama and Nilakantha of Tirur, among others.

Madhava's (1340-1425 CE) work on calculus is said to predate the work of Newton and Leibniz. We do not know what Madhava actually looked like, so this sketch is only indicative.

Research on this alternative history of mathematics has conclusively shown that the work of the Kerala school predates the discovery of that great mathematical tool, 'calculus', by at least two centuries! While Newton and Leibnitz, generally acknowledged as the founders of calculus, must be given their due for combining a range of disparate ideas into the coherent discipline of calculus, the discoveries of Indian, Chinese and Arab mathematicians cannot be disregarded.

As Joseph pointed out, we cannot overlook the transmission of mathematical ideas from Egypt, Babylon, China and India through the Arab world to Europe. Research in this area has thrown up compelling evidence that this mathematical transmission, right from the days of Pythagoras, of antiquity, had informed the development of European mathematics. Pythagoras knew that the Egyptians knew his theorem, and he also knew they hadn't proved it. Perhaps they did not need to, since their focus was on the practical uses of mathematics.

Thanks to my chance encounter with Joseph in the mid-1990s while I was a teacher, I could further explore the seed of curiosity that Channa had sown in the mid-1980s. I faithfully shared these exciting discoveries with my students. Also, I ended up buying Joseph's book *Crest of the Peacock*, in which he elaborates on the theme of mathematics done and discovered outside of Europe. The book starts with the mathematics of the 'Ishango bone' (linked to a lunar calendar) from the mountains of Central Equatorial Africa, which proves the existence of mathematics at least 20,000 years back! It is a fascinating read and will certainly open your eyes.

I'm not sure of Channa's opinions about these alternative perspectives of the history of mathematical development in the world. In any case, we didn't discuss it in school. History in general is complicated, interesting and slippery. So is the history of mathematics.

The chaps who taught me in the eleventh and twelfth, and in my four years of engineering, were committed teachers, no doubt. But they were committed to teaching only with an eye on the examinations. No stories, no perspective, no flights of imagination. Just mechanical teaching to finish the syllabus. On top of that, they told us rather ominously, "If you don't get such and such percentages, you will not get into engineering or medicine."

As if these were the only two worthy things one could do in life.

It is one thing to experience the joy of acquiring knowledge. But there is also the question of what you do with that knowledge. For a while I entertained thoughts of becoming an astronomer or a theoretical physicist, for I felt that physics was a 'deep' science. The idea of dedicating one's life to creating and discovering knowledge about the natural world was alluring, but the social aspect kept coming back to me again and again. I decided that making that knowledge accessible to everyone would be a more worthwhile pursuit. The task of exposition was far too important and needed urgent attention.

The adventure continued beyond The Valley School. It had to, because the reasons were deep.

6 My logbook: learning about education

“So what,” they said, “why do you want to leave?”... “Children are children everywhere,” they emphasised. I agreed. “But can we close our eyes to this unequal and unfair education system?” I asked.
– a conversation at the Valley, 1995

I decided to leave The Valley School in 1995. Not because I didn’t enjoy being there. Far from it, in fact. But something kept gnawing at me. Much as the school espoused an education that kept the child’s best interests at the centre, I felt we were cut off from the rest of the world. Our well-endowed, beautiful and wild 100 or so acres outside the city was like an island in a large ocean of schools dotting the landscape, including those in our immediate neighbourhood. Many of these were run by the government. They spoke of neglect. Often they did not have enough teachers, their walls were dilapidated and they looked depressing.

I felt a deep urge to do something about it. Sometimes, I wondered what Channa would have done. Much to my disappointment, not many in the Valley were willing to engage with this question. “Let us set our own house in order,” they kept saying. That sounded like going around in circles, for there is no such thing as setting one’s house in order. Something always remains to be done.

Day after day, I would see a group of children walking barefoot across the valley to reach their middle school at Thatguni village on the other side. Most of them were from Rachenmada, another neighbouring village that only had a primary school. Once I had even taken my seventh graders to Rachenmada. For many of them, it was their first-ever visit to a village. During my second year at the Valley, we took a small step and invited the students and teachers from the Thatguni and Rachenmada schools to our Science Day celebrations. They liked what they saw, including the mysterious π tail. That is what they said. I was very happy.

I began to realise that there is a deep-rooted inequality in the way our society educates its children. Here I was, working in a school that tried to give so much to its children, most of whom came from well-to-do households. Yet right next door we had schools that looked and felt so different. The difference couldn't get starker. I worked in a school whose concern it was to harness the child's potential to the best possible extent. Outside our little island, there were many, many schools run by the state that were indifferent to the potential that lay hidden within each child. And then there were many other schools where children were mindlessly driven to get good marks, their passport to what is defined as a 'better' life. I too had graduated from one such school. But Channa had made a difference there.

So, I decided to leave The Valley School. I felt I could be more useful in schools with bare walls and broken furniture, schools without teaching materials, schools where children from poor families went and learnt by rote, or did not learn much at all, or went because they did not have anywhere else to go. My colleagues said, "So what? Children are children everywhere." I concurred. But my question was: Can we do something about this unequal and unfair education system? Can we afford to ignore it?

I continued my adventure in education, gratefully acknowledging the foundational years I had spent at the Valley. My first pit stop after leaving was a remote block of Raichur district in northern Karnataka. There, I attempted to work with 60 government primary schools to improve the way they taught maths and science. Then came a quantum jump of sorts. I got the opportunity to work at the national level in a government programme that attempted to universalise primary education in the country, meaning providing free education for all children till the primary level.

Our forefathers who wrote the Indian Constitution had promised that every child in the nation would be provided free and quality education till the age of 14 years. However, the difference between rhetoric and reality is harsh. That's when one discovers there are millions of children who cannot go to school. They look after their siblings, work as rag-pickers, survive at railway stations, get abused, make firecrackers, use their nimble fingers for carpet-weaving, work long hours in roadside dhabas and the like.

We, in our middle-class cocoons, tend to take schooling for granted, as if it is something that happens naturally in everyone's life. But we need to open our eyes to what is happening around us and train ourselves to look at things more finely. I did that and saw stark inequality.

In the Valley we grappled with the individual child, and we grappled with ourselves as adults who played a role in the child's life. We tried to examine our beliefs, fears, biases and saw how they influenced our interactions with each other and with children. As I started working with the larger education system, I learnt that larger social issues had to be urgently addressed which prevented the full blooming of a child's potential. I realised the intimate links between the individual and the social. The social demanded urgent attention if education had to be egalitarian.

I moved on and did a long stint with some international organisations that were into child development. My work in education continued through advising these organisations, helping them develop their perspectives and their educational programmes, and so on. Then came a period when I left my institutional affiliations and became a freelancer. A number of exciting and enriching assignments followed, across the country. I worked with a number of organisations – some big, some small – did education research, developed training programmes, travelled a lot and generally enjoyed the freedom of being on my own. After five years, I returned to a full-time job again.

As I delved more and more into the nature of the government schooling system, I used my experiences from the Valley as a reference point. They offered a useful, though at times unfair, comparison. In the Valley, teachers came on time. In the schools run by the government, you cannot take this for granted. Many teachers come and go as they please. In the Valley, you could say that most teachers were highly motivated and did not need any monitoring. In the schools operated by the state, this was not the case. Here inner motivation was an issue, save for a minority of teachers who needed no telling.

In the Valley, children did not drop out. In the government schools they did, class after class, especially the girls and children belonging to very poor homes. The schools drove them out.

In the Valley, parents were closely involved with what was happening in their child's life in school. One could not say this for the schools run by the state. Here a different dynamic operated between parents and schools. 'School management committees' were mostly just on paper and, barring a few exceptions, were disconnected from the schools. The parents and teachers tended to blame each other for the child's failures. I can go on and on…

I started confronting the challenges of making the government work in education after moving out of the Valley. There was this huge, multi-layered 'bureaucratic system' in education that extended from the national level all the way to the school. Our challenge was to make this system capable of addressing every child's educational needs. Moving the bureaucracy was often like getting an elephant to do a U-turn! But it continues to be a fascinating aspect of my work.

In education, the teachers are the foot soldiers. I quickly realised that the bureaucracy did not trust the teachers. The administrators like to control them. Teachers are told what to teach, how to teach and what else to do. There are elaborate training programmes to instruct them. Crores of rupees are spent in these efforts. In this rush to equip them, I sensed little effort to evolve and implement a long-term vision of teacher development. Much of the time we are running hard to stay in the same place.

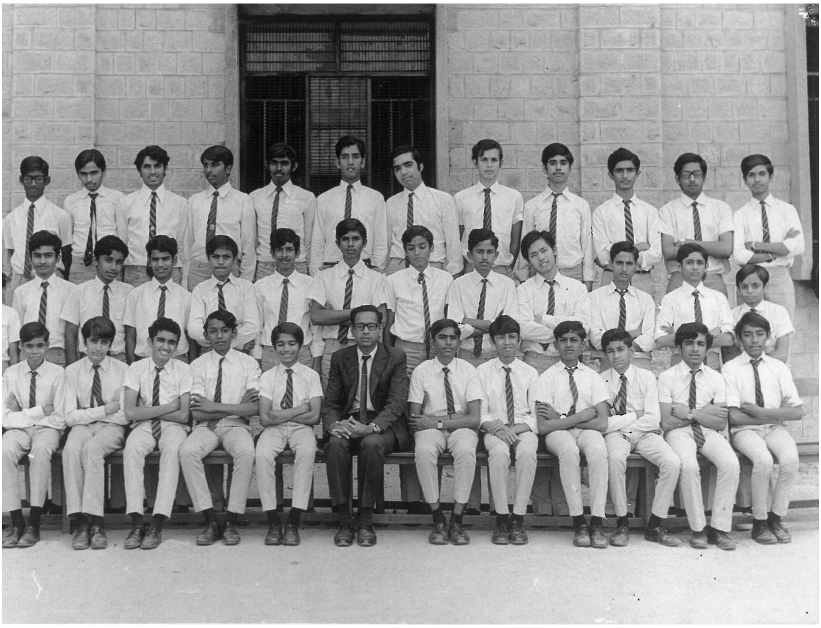
Over the years, I realised that we need to do much more than train teachers. They need an environment in which they can grow as professionals and as human beings. They need opportunities to learn from each other, to see what is happening elsewhere, to read, write and share their ideas. Most importantly, they need to be mentored and supported in their growth.

I realised that Channa's story could be my small way of giving teachers their due, by highlighting the need to look at them differently and also learning from what great teachers do. Remember, the subject, its craft and the teacher are all intertwined. We need to travel to all these places.

But along the way, another question cropped up: Am I the only one, I wondered, who feels about Channa this way? So I thought, let me see what others have to say about him. This led me to some interesting conversations.

Memory can play strange tricks. And here I am, dealing with multiple memories across time and space.

Channa giving a lecture at Baldwins

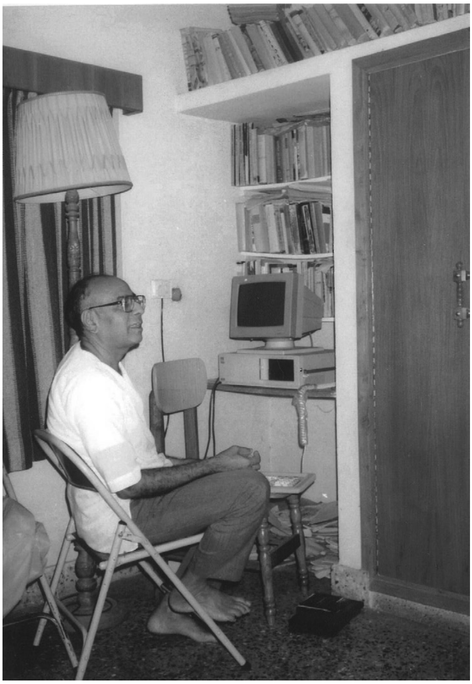


Channa and his students, batch of 1972-73, in front of Lincoln Hall

With students and staff at Bethany Junior College in Bengaluru

Channa with students on a school excursion to Nepal

Staff photo, 1964. Standing (L to R) – Channa, Pasha (Physics teacher), Suputhra (Biology teacher), Wilson (Chemistry teacher), Nehemaiah (Hindi teacher); Sitting (L to R) – Jeevaneson (History teacher), Gershom (Geography teacher), Finch (Principal), Reverend Heins and Mrs. Heins (English teacher), Paul (English Teacher), Pakianathan (Physics and maths teacher)



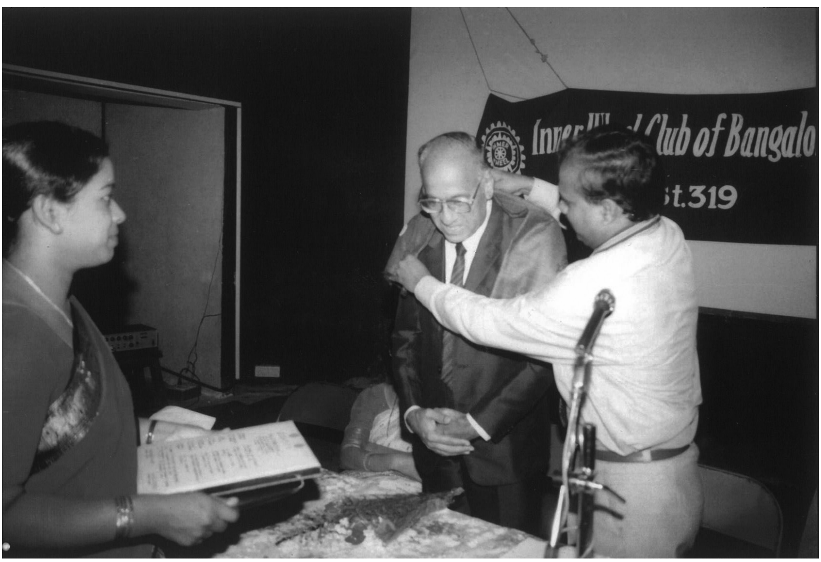
Channa in his study. Note the books, arranged hither and thither

Family photograph -– Sitting on the Diwan (L to R) -– Channa's father, Channa with Grandson Suveer, Channa's elder and younger sons-in-law; Sitting on the ground (L to R) -– Sharada, Channa's wife, and their daughters Rohini and Ranjini

Channa in an animated conversation with Sharada in their little garden

Another view of Channa's home in Mattur in Shimoga district, Karnataka, now a Vedic Pathshala

Channa, Sanna and Danny Soanes on the excursion to Nepal



The Rotary Inner Wheel of Bangalore honours Channa

"The month of November is traditionally the month of thanksgiving here in the US. I would like to take this opportunity to thank you for being the BEST math teacher I have ever had!! You taught with interest, knowledge and compassion and, thanks to you, I developed a healthy love for the subject though I am not an accomplished mathematician."

– Kishen Bhagavan
Class of 1974

"You taught me mathematics at Baldwin Boys High School many years ago. I passed out in 1974 along with Kishen Bhagavan and others. I got your e-mail ID from Kishen. I know this letter expressing my gratitude is a little overdue, by about 35 years! I simply must let you know that I feel so privileged to have had teachers such as yourself who taught us the real value of the subjects we learnt.

Although math was never my strong subject, I can still do simple calculations faster than most because of the techniques we learnt in school before the age of calculators.

Thank you Sir, for being such a wonderful presence in our lives. I do hope you are well and happy. I live in Mumbai now and I am a playwright and stage director. I am told you are in Bangalore. It will be wonderful to hear from you."

– Mahesh Dattani
Class of 1974

"By the way, do you know how old he is now?"

– Biju Sam Jacob
Class of 1984

7 The beholders' eyes

"God broke the mould after he made
MV Channakeshava."
- Rohan Joshi (Class of 1976)
"They don't make them like him anymore."
- Ravi Ramu (Class of 1976)

"Channa was very aloof as a person, but he had a high level of integrity. He was a clean individual." This is what Kishen Bhagavan from the class of 1974 told me on the phone one day. He was the first alumnus I spoke to after I decided to get in touch with students taught by Channa, both senior and junior to me. I had reached a stage in my writing of this book where I was curious about moving a little away from the maths, to look a little more closely at Channa the person, through the eyes of his students. What did they think of him? Were their observations similar to mine?

When I began my journey of writing about Channa and his teaching, I wanted to explore the world intimately linked with him through the myriad thoughts and opinions of his students. I surmised he must have taught thousands of Baldwinians during his 36-year tenure in the school. A rough calculation would put the number in the region of 5,000. What did these 5,000 Baldwinians think of him and his teaching?

It became important for me to capture the essence of this world. I saw it as integral to this book on Channa. We cannot separate the person from the teacher – we must see the totality. I needed to contact Baldwinians, young and old, all the way back from the 1960s up to the 1990s, who could throw light on Channa, the person.

The early leads were provided by Channa himself. He shared e-mail addresses of three students who had kept in touch with him over the years. He mentioned a few more names. I began writing to my fellow Baldwinians and the word gradually spread thanks to Facebook, that great networking tool. Pretty soon, I was in touch with several students whom Channa had taught. They populated professions ranging from theatre to teaching to technology to investment banking, and they were located in different parts of the world. Several of them had become dynamic entrepreneurs.

I also discovered that there was a vibrant Baldwins alumni community online. Goverdhan Jayaram, an army major from the class of 1976, introduced my book-writing project to this community. When I met him over coffee, he said, "Look, I was never interested in math. In that sense, Channa didn't have an impact on me. I wanted to join the army and I worked towards it from the beginning. But I can help you with the book by connecting you with other Baldwinians."

Following Goverdhan's introduction online, I placed requests for people to respond to a few simple questions I was asking about Channa. Interested Baldwinians wrote back. I imagined that many would respond but was then a bit disappointed. Anyway, I am grateful to the ones who did, taking time from their hectic lives for something that was close to my heart. There were phone calls, e-mail exchanges and some face-to-face meetings – enriching stuff that warmed my heart.

In November 2013, the Baldwin Alumni Association organised for the very first time a 'Baldwin Career and Business Fair', an event that attempted to bring past and present, young and old Baldwinians together. It was an amazing idea – bringing ex-students from all walks of life together on a single platform to share their life experiences with their young counterparts about to graduate from school. The fair also sought to enhance business opportunities within the Baldwins family.

I met many interesting Baldwinians including some of my classmates, after some 28 years! The event got me more contacts and I began communicating with them about Channa. Their reflections about the man and his teaching were warm, interesting, often touching and insightful. They helped open up facets that went beyond the maths he had taught us. Each person I met or spoke to remembered Channa in different ways.

In this diverse mix of memories, I could trace common threads that resonated across space and time, reinforcing each other. I could easily make the connections with my own observations every time a point was made. Those three years of school came alive.

Unnikrishnan (class of 1988) brought back memories of Channa's fastidious formal attire:

> "I can't forget his brown suit, thin and long tie and neatly ironed white shirts. Bespectacled, he would walk trim and proper into the class."

Unni and I met at the Baldwins Career Fair. We squeezed ourselves into a tiny corner just outside the entrance to Lincoln Hall, the impressive stone building where the school usually holds its activities like debates, drama, elocution, prize-giving ceremonies, special assemblies and the like. During my time, the ground floor housed the chemistry lab as well as the eighth grade where we first met Channa. The career fair was being held in Lincoln Hall and Unni was an invitee, like me. He had come there to share his experiences in the world of finance with the graduating students.

As Unni talked, I remembered Channa's Lambretta scooter, which he had bought from someone that Pasha, his colleague and our physics teacher, had recommended. Then there was Sanaulla, the Hindi teacher. The jodi of Channa and Sanna, as they were known. Channa, Sanna and Pasha were the three musketeers on their iron horses – as they themselves saw it.

Suresh Menon, 13 years Unni's senior, echoed his description of Channa's attire:

> "Memory can play strange tricks sometimes, but I remember Mr Channakeshava was usually in an off-white suit. He stood out in the school photographs in the 'staff' section among the predominantly dark-suited!"

As I busily scribbled, holding my notebook in my left hand for want of a support, Unni reminisced with a faraway look in his eyes:

> "He surely didn't scare us away, like some of the other teachers. He didn't treat us like kids. He would never use the word 'children' when addressing us... he would always say 'gentlemen' or 'young men'. I felt respected."

I couldn't agree more. As I looked for more responses from fellow Baldwinians, Kishen again wrote saying:

> "Channa was a serious person. He left very little room for us to tease him and thus get close to him. But he was dedicated and kept very strict control of the class with his serious approach... I do not remember him kidding with us even by accident... for us to tease him or get close to him. He had a serious demeanour and made sure there was no deviation from this. All we could get out of him outside the class was a simple acknowledgement."

I went back in time as I read Kishen's reflections again and again. He was spot on with his observations. There was something about Channa that quietened us and made us focus on nothing but mathematics. Maybe it was his command of the subject and the passionate way he dealt with it. You just wanted to listen to him and didn't want to miss out on anything he said. What Kishen labelled as 'control' was for me the ability of the teacher to get us to focus willingly on the task at hand, of learning. It was not negative control by exercising fear and punishment. With the kind of respect he commanded, Channa did not have to resort to any other means of control to manage us.

From faraway California, Biju (class of 1984) shared his portrait of Channa:

> "He was strict yet cheerful and always pleasant. He always had a smile. But few students dared to play the fool with him."

Like Unni, Biju also felt that Channa treated students as mature young adults. Once when he and his classmates went to Channa's house to get some doubts cleared, he offered them sweets since it was Diwali time. This act of friendliness surprised them, for they were not used to teachers being warm and friendly towards students.

I too felt this formal distance at Baldwins, a regimented school with strictly-defined boundaries. The hierarchy between us and the teachers was rigid. We didn't get to know them better. All of this changed when I became a teacher. At The Valley School, the teacher-child relationship was more informal and warm. It was a huge shift for me, coming as I did from a regimented system.

Though Channa had a serious disposition, he often enlivened his teaching with humour, as Rohan pointed out in his portrayal:

> "He was not opposed to humour, leg-pulling and the like. He was a sport. In one of our classes on trigonometry, he drew something on the board and asked me what could be the profession of a person who builds it. I said, 'Undertaker.' He laughed heartily and said, 'Sorry to disappoint you, Rohan, that was a yacht. So much for my drawing skills!'"

Joydeep Sengupta (class of 1984) put it succinctly when he observed:

> "Channa had a wry sense of humour that got his students comfortable with numbers."

This was interesting. Humour can play a critical role in promoting both learning as well as a feeling of well-being in children at school, which we haven't understood well enough.

There are other Channa snippets. "Be intelligently lazy," was Channa's cryptic message in Biju's class one day. What Channa meant was that you need to work in such a way that you don't have to do more later. These three words have stayed with Biju to this day. He follows this advice when he develops software, administers computer systems or designs algorithms as part of his job, or even when he indulges in his hobbies of electronics or automobile repair. For instance, he spends time to automate computational processes so that he does not have to repeat the same operations. "I attribute this philosophy to Channa, this being lazy but in an intelligent manner."

Biju recalled another well-known Channa statement, delivered with a smile, almost mischievously. "Don't put the cart before the horse," he would say, and then wait for the inevitable question about what the idiom meant to pop up. I too had heard this. While we can find our own meanings as to why Channa used this statement, it continues to help Biju get the foundation of any computation system or model he is working on right, before he builds other elements into it.

That's the sort of influence a good teacher can have on you even years later. In Biju's case, though the maths "couldn't get inside his head" at school, he ended up doing his master's in mathematics. "Mathematical probability gone awry, just in case you tell Channa about it," he cautioned me as we exchanged e-mails. "Hold him firmly lest he falls off his chair when he hears this."

Everyone I came in touch with echoed Rohan Joshi, who asserted:

> "Channa loved teaching math. He was born to teach and he loved teaching, which also explains why when corporal punishment was rife in our school, he totally eschewed it. I cannot remember him ever using the cane. Though we had a few other senior teachers who knew their subject well, they mainly taught through fear. Channa was head and shoulders above the others."

And Kishen further added:

> "There was much capital punishment those days but he never laid a finger on any of us even once and he never belittled us. Baldwins was a very regimented and authoritarian school, which instilled fear in us."

I nodded in agreement. Channa worked in an institution whose teachers, by and large, were conditioned to teach through fear and control. Corporal punishment was resorted to easily and the teachers saw it as an effective tool that could be used in the act of teaching. Channa must have had strong convictions not to resort to corporal punishment.

Fundamentally, he was, as Sridhar Chintapatla (class of 1970) said, "A very gentle soft-spoken human being who saw everything from a human angle." Even mathematics, thanks to his teaching, was humanised. That is what I liked about his style.

"Channa was a 'gentle' teacher who was far from intimidating and always keen to help you learn," Joydeep added. Abraham Cherian (class of 1984) augmented the same with a sensitive comment:

> "My first impressions of Channa were that of a mild-mannered school teacher. Very soon, it became apparent that he was one of those teachers who could control the class without having to raise his voice... a rare quality I must say."

> "There was something avuncular about him," said Suresh Menon. "But he kept his distance and was clearly not 'one of the boys' like a couple of other teachers were."

He was more than gentle and mild-mannered, as Navam Pakianathan (class of 1984) pointed out:

> "Mr C was a calm and cool person. I never saw him lose his temper or raise his voice. He would certainly get angry with non-compliant students, but he would speak to them in the same volume, with added sternness. He was always on time to class and had high expectations of us in keeping attention and in note-taking. He was very consistent in his emotions."

There were other facets to Channa that I began discovering during my interactions with my seniors and juniors. Kishen shared his experiences from a class excursion in 1974:

> "There was this school trip to Ooty. Channa along with some other teachers accompanied us. One of the teachers got very drunk during the trip, shouted at me and hit me on my head. Channa saw how humiliated I felt in front of everyone. He took me to sit with him for the rest of the trip to ensure that I didn't have to face the brunt of this drunken behaviour again. Though he didn't say much, I felt very touched by his gesture."

After moving to the US, Kishen tried hard to get in touch with Channa. Finally after posting him a letter, he received an e-mail from his tech-savvy teacher!

For some reason, as I kept reading Kishen's narration of his trip, I remembered how Channa took some time during my

ICSE exams to provide me with the emotional support I needed, for amma was in hospital, battling extremely high levels of blood sugar that had made her condition critical. Those few minutes that he spent with me were enough to put me at ease and calm me down. Also, in grade 9, when I almost failed to make the grade in the maths final exam, an anxious anna was asked to come to school to meet Channa. Patiently putting his arm around anna's shoulder, he told him not to lose hope and assured him that I would soon improve. I did.

Giri Balasubramaniam (class of 1986) fondly remembered how Channa once called him home to counsel him when nothing seemed to go right for him in his maths-learning in the eighth grade. What started off as a counselling session ended up as regular tuition, and Channa did not even ask for a tuition fee.

I'm sure there are myriad stories of Channa that each one of us can recall. But this classic Channa story by Ravi Ramu, Kishen's classmate, takes the cake. It is worth reproducing in full:

> "This story of Channa will be with me forever. The year was 1974. We were the outgoing students of Class 10A, ensconced in the ground floor of the Lincoln Hall building, in Channa's class. He was, indeed, our class teacher. One of us (no, it wasn't me) left a 'Lakshmi' bomb in Raja Jumani's desk. The bomb's wick was tied to a lighted agarbatti. The whole class then departed to the physics lab, Mr Pasha's class. The lighted agarbatti finally reached the bomb's wick and the explosion happened in the twinkling of an eye! Lincoln Hall and the whole building reverberated as though the Pakistan war of 1971 had started all over again in 1974!
>
> The next day, Channa was charming! He was charming but incisive. In his trademark tranquil and taciturn manner, he proceeded to give us, Class 10A, a dressing down followed by a lesson that is still firmly embedded in my mind. "You are men," he began. "Men in their final year at school, on the cusp of embarking on life's journey. Yet you act like children, afraid to own up to a churlish deed that the whole class should be ashamed of!"

> Us 43 'boys' turned into 'men' that early 1974 winter's morning. Such was the manner in which Channa could turn a dastardly deed into a masterclass of life. Hats off! Only Channa could have delivered thus.
>
> They don't make them like him anymore!"

Channa, like Ravi says, could win you over with his words.

When Unni's class was being caned over an allegation that some of his classmates had vandalised their classroom, Channa pleaded with them, trying to bring in a more humane approach to deal with the incident. *"Tell me please, if you know," he said. "Let us put an end to this (punishment)."*

Our class had its own cracker episode, when we were in 9B. An 'onion' bomb went off in the class during the English period. It resulted in equally violent outbursts from our teachers, followed by a public apology that a classmate of mine had to tender during assembly time. As the principal read out the apology on my classmate's behalf, my classmate imitated him. It all felt very amusing and some of us even laughed at the irony of it all.

Stories about Channa abound, as you can see. Let me continue telling them. When Vijay Kumar Bommu graduated from school in December 1969, Channa wrote in his autograph book which Vijay has preserved to this day: "May your life be a glorious success."

Vijay recalled the epochal moon-landing of July 1969, when Channa stopped teaching maths for a week to discuss the event every day, explaining everything about the Apollo mission, about NASA (National Aeronautics and Space Administration), the rocket, the fuel and other details to his rapt students.

> "Those days, we could hear the conversation between ground control and the crew on AIR (All India Radio) and BBC (British Broadcasting Corporation). It was a great experience."

> "Channa was very accessible. You could easily go and talk to him. He was willing to listen. Because of him I got interested in math."

I connected what Vijay said with an observation of Rohan's, that Channa could have been a great physics teacher as well.

Channakeshava, the gentle and mild-mannered but firm teacher. The cool and calm teacher. The caring teacher, who respected you and who thought of you as a mature young adult. The teacher who never raised his finger, nor his voice, yet commanded your complete attention and respect. The conscientious but serious teacher who was open to leg-pulling and appreciated wry satire. The teacher who was passionate about mathematics, who lived, breathed and taught it.

"They don't make them like him anymore!" Didn't someone say that?

Channa joined Baldwins in 1964. In October that year, the Government of India commissioned, for the first time after Independence, a report on elementary school education in the country. The exercise came to be known as the Kothari Commission Report, named after Daulat Singh Kothari, a physicist who was then the chairman of the University Grants Commission. The Kothari report is quoted widely even today.

"The destiny of India is now being shaped in her classrooms." So began the first chapter in Volume 1 of the report, which took all of two years to be written. What was striking about it was the definite links it made between what was happening in classrooms with the goals and challenges of national development. It called for a clear departure from traditional ways of teaching and learning, and firmly stated that knowledge cannot be received passively by the learner. On the other hand, it needs to be actively discovered by students and teachers alike.

"To know," the report said, does not mean "to know by heart". The task of education is not to merely help the student develop his or her skills of collecting information, but to develop creative and critical abilities of thinking. This requires a revolution in education without which, warned Kothari's report, "... students in schools and colleges will not be able to receive the type of education needed for a new society."

In a small corner in Baldwins, Channa was already at work even as the Kothari Commission began writing this magnum opus on the state of school education in independent India.

8 As they saw it

"We don't have to study anything, Sir. You just listen to him and go home."
- What the boys told principal Reverend Gokavi about Channa in 1975

I often wonder what it was about Channa's teaching of mathematics that stayed with me for so long. When I first wrote the article about him, I surprised myself by recalling minute details and examples from his classes.

Channa developed his craft with love and care over the years. He had sure-shot ways of ensuring that even the most maths-phobic among us would sit up and listen. Take me, for instance. I was an average maths student. Channa will no doubt agree. But here I am, celebrating my maths teacher and his craft. Something changed in me, forever. That happens when you have a teacher like him.

How did his students see his teaching? Was I the only one who felt this way? Let me capture Joydeep's earlier quote fully:

> *"He had an innate wisdom that helped arouse the curiosity of his students, and a wry sense of humour that got his students comfortable with numbers."*

I believe the 'innate wisdom' that Joy talks about came from Channa's love of the subject and his passion for communicating its excitement. These are two essential ingredients for any good teacher. How does a teacher combine these ingredients while teaching? I guess the path must be found by each individual. But I can tell you that the Indian state makes it all the more difficult for this to happen. There is simply no environment that makes teachers like Channa.

Abraham Cherian takes it further:

> "While it is over 30 years since I first met him, I could tell that this was a person whose love for mathematics ran deep. It was not just his knowledge of the theorems, equations, etc. He knew so much of the development of the subject."

There you have it – Channa, the eternal student of the subject he teaches and knows so intimately. He knows where it has come from, its trajectory. And when he teaches, you embark on a great adventure. No wonder then that Channa was as much a history teacher as a maths teacher. I think it was this perspective, this vantage point he had about mathematics that turned out to be his strongest ally in teaching.

But having this vantage point is not enough. You need to find ways of communicating it so that a child picks up your train of thought. Channa could not only pose questions but also answer the many 'whys' that we constantly threw at him, in a language that we understood.

Being keenly aware of his own learning experiences, he recognised the value of making us think through problems and arrive at our own conclusions. Rohan describes this beautifully:

> "Channa made us THINK, and took the angst of math away from me. At an impressionable age when math was the bête noire of all concerned, he was unapologetic and made sure that everybody understood the fundamentals."

And here's what Kishen added:

> "He didn't 'teach' us as such but challenged us to think on our own."

Tejas (class of 1996) complemented Kishen:

> "Unlike many of our teachers who were more focused on forcing us to memorise information and did not appear to enjoy teaching too much, Channa was a contrast. He loved math... in modern parlance, we would call him a nerd, not in the negative sense of the word. Channa enjoyed mathematics himself and that is why he enjoyed teaching the subject. He always threw problems at us that made us think."

Isn't that interesting? Teaching to make students think is vastly different from teaching to share information and facts, or teaching to help students pass exams. Getting students to think on their own and embark on an intellectual adventure, while at the same time preparing them for the exams is a tough balancing act that Channa managed to perfect. He showed that exam preparedness did not have to be at odds with understanding and appreciating the subject. The wonderful thing was that he let you discover things for yourself, only occasionally nudging you in the right direction. While that nudging was important, he never provided you the answer. That, you had to discover on your own. Above all else, Channa valued individual effort.

Ah! Teaching children to think. Perhaps that's the most difficult task for an adult. Most of the time, we exhibit two extremes of behaviour because we are impatient. We either scream in disgust and brand them incapable of learning, or we supply the answers too quickly because we don't care enough to let them develop the ability to think for themselves. Watching all that stumbling, getting up, stumbling again is too painful, eager as we are to prepare them to score high in exams and just go with the flow. In both cases, we are doing a great disservice to young minds that are setting out to explore the world.

Channa was unapologetic about getting everyone on board. For him, being in the moment was important. He was not worried about where you went eventually but, at that moment, you had to savour what he served up about the subject. This applied to every one of us. In this sense, he was democratic in his approach, not believing for once that nothing could be done for students who didn’t take to maths easily. I suppose this distinguished him from all the other teachers who taught us. He was infinitely patient.

When a teacher is so sure of his subject, confident and caring enough to get children to grapple with it, no other tools are needed to capture everyone's attention. No other means are needed to manage the class. Which is probably why Channa eschewed corporal punishment at a time when it was rife at Baldwins, something that all his students pointed out. Rohan’s statement says it all:

> “I cannot ever remember him using the cane. He did not rule by terror but by knowledge and humour.”

Channa used his sense of satire to keep our attention and relieve our fears. Yes, fear was often the underlying text of our interactions with other teachers, who resorted to the cane at the first instance. Channa’s sense of humour was quite sophisticated, as Abraham Cherian once discovered. He told Abraham to ‘turn the book around’ and look carefully at the geometry problem he was struggling with. And lo, there was the answer! Not that Channa advocated turning the book around to get the right answer. It was just that some problems required out-of-the-box thinking to view them differently, to find their solutions.

24 years after Rohan graduated from Baldwins in 1975, there was no change in the fundamentals of how Channa went about teaching maths. Gurumurthy (class of 1999), perhaps from the last batch to be taught by Channa at Baldwins, observed:

> “Though math was not a favourite with every student, there

> is this underlying agreement that his style of teaching was gripping and there was never a dull moment. Making us all mathematicians was not his goal, but ensuring that we did not get put off by it certainly was."

Interestingly, Guru was taught computer programming by Channa, who handled the subject in his last four-five years in the school. Even here, Channa made them examine problems and encouraged them to come up with their own solutions, or in other words, to think and derive things on their own.

Kishen's comments throw some light on the kind of school Baldwins was during the 1970s, during the time his class was taught by Channa. A key feature was that there was not much discussion in class. You just had to do as you were told. But as he observed, Channa surprised him, as he did many of us:

> "Teaching at Baldwins was traditional, rote. The emphasis was on mugging rather than conceptual understanding of the subject. The focus was on routine and regimentation rather than creativity. The teachers, in general, were men who probably found themselves in the profession as a last resort. The pay was not very good either.
>
> "How Channa managed to be creative in this atmosphere is a mystery. I think it was his love for the subject that brought it out."

Channa's teaching hinged on his emphasis on the fundamentals of the subject. Once we got the hang of the basics, we were able to get along happily. Knowing his subject so well, he was able to pinpoint exactly what was needed to develop an appreciation for an idea or concept. As Rohan elaborated:

> "Channa was a first principles person. He reduced everything to the absolute basics. Algebra is a difficult and abstract subject, geometry and trigonometry have their own complexities and arithmetic is absolutely fundamental. His treatment of the

various branches of maths, explaining to us where it is useful in our future lives, was the basis of his teaching.

"Channa made us see the application in real life of what he was teaching us. When I went sailing for the first time in 1999 in the Netherlands in my friend's sailboat, he asked me how I could so precisely net the sail to the mast. I said very casually it was the result of first principles of trigonometry taught by my grade school maths teacher."

As much as Channa emphasised the practical side of mathematics, I also saw him recognise the inherent beauty of the subject. It was this recognition that made him dig deep into its reservoirs and show us some of the treasures that lay within.

Because he was a first principles person, the process was as important as the product, as Suresh Menon recalled:

"He was the first math teacher who gave you marks for getting the steps right even if the final answer was wrong. The process was as important as the product. At the same time, the English was as important as the math!"

Biju recalled discussions with Channa that brought out his sophisticated understanding of the subject. This is how it went. Channa posed a simple question in an introductory statistics class one day: *"Suppose Sir CV Raman (our Nobel Laureate in physics) makes a statement that Bangalore has the best climate in the world. How does one prove this, one way or the other?"*

Unlike other examples, such as 2+2 = 4, which is right, and 2+2 = 5, which is false, how does one see CV Raman's assertion?

An interesting discussion ensued. Channa's point was that this is quite different from the proofs that are required in geometry, for instance. There, one proves theorems one way or the other. The answer is either a 'yes' or a 'no'. It is like saying 0 or 1 (0 standing for 'yes' and 1 standing for 'no'). But how do we engage with Raman's statement?

Channa pointed out in Biju's class that large amounts of data on climate and the factors determining the climate of Bengaluru and other cities are needed. The altitudes, maximum and minimum daily temperatures, distance from the sea, etc, need to be recorded. Further, this data has to be taken for 365 days of the year for at least 50 years to observe its trends. You need to classify, tabulate, analyse and interpret this voluminous data to draw conclusions from it.

Doesn't this sound a bit like our arguments with Channa about measuring the three angles of a triangle? There, Channa had us stumped when he asked about the 1001st triangle. But here, collecting a large volume of data is what you need to do. You cannot prove or disprove Raman's statement without doing so. This was also what was done in the case of the Four-Colour Problem, though it led to great debates about the validity of the method as a mathematical proof.

Channa told Biju's class that in statistics, which is also a branch of mathematics, the questions we encounter cannot generally be answered by a simple 'yes' or 'no' because it is here that one encounters the notion of 'probability' and 'degrees of certainty'. While the methods to find the proofs of problems may be either deductive or inductive, in statistics the probability of an event lies between 0 and 1. The weatherman has to live with probability every day. So does the physicist. And that is how Channa brought out another layer of understanding to the discussion on proofs which we encountered earlier. What was thought-provoking for the students in Biju's class was the range within which one could claim certainty.

Let's move on. I have something to say about how Channa used the blackboard. We often use the term 'chalk and talk' to negatively connote a teaching practice that is limited in its scope. It implies that the teacher only lectures the children and scribbles something on the blackboard without doing anything else. We say this sort of teaching is boring, that it does not provide us the opportunity to learn through our experiences of the real world, which is required in disciplines like science and social studies.

How much does this apply to the teaching of mathematics? A good teacher can really use chalk and talk well, and take us on a great adventure of the mind. Chalk and talk can weave magic. In this case children do not necessarily experience the objects of the real world but struggle with ideas and concepts. They enter another fascinating world and experience something altogether different.

Channa's use of blackboard space and his impeccable handwriting were unique elements of his teaching. I have not come across another teacher who used the blackboard as effectively and as beautifully as he did. His handwriting was a visual treat, not flamboyant but clear and pleasing to the eye. The sense of proportion he had when using the blackboard was amazing. He would clearly demarcate areas on it – for diagrams, for arguments, for steps of an analysis, and for rough work or calculations.

I tried to perfect this craft as a teacher. There were many occasions when I would slip into a classroom well before it began. I would then adorn the blackboard in my best handwriting, planning the blackboard space to the best extent I could. And then I would enjoy the appreciative expressions of the children as they came into the room and settled down for my session.

Navam, one year my senior, who has been teaching physics for the last 20 years, had similar sentiments to share:

"His systematic way of teaching mathematical concepts and theorems stands out. His meticulous board work and diligent preparation for class made him a great teacher. I personally imbibed his ways of teaching. For the past 20 years I have taught physics with board work as neat as I can manage. This is in large part due to the influence of Mr C in my life."

For me, Channa will always remain the model maths teacher. He perfectly straddled the discipline of mathematics and its teaching. He was its passionate student, while at the same time, he loved teaching the subject. It is widely recognised in educational theory that the teacher's knowledge should inlcude both subject matter knowledge and the knowledge of teaching ('pedagogic knowledge'), among the aspects of knowledge needed to become a good teacher. The challenge of teacher education is sometimes seen as a knowledge problem. In a class of nearly 50 students and in a school that was traditional in approach, Channa struck a great balance, for his teaching is remembered till today.

The teaching of procedures in mathematics ('procedural knowledge') is only a small part of mathematics education. We are repeatedly taught to 'carry over one and borrow from the other'. We follow standard steps to do multiplication and long division, and there are procedures that one follows to get the 'right answer'. In doing all of this, we may not ask why we follow a certain procedure or why it works. Most teachers see mathematics as a set of procedures that need to be memorised. It takes a good teacher to show that there can be several ways of getting at a problem. But this approach is usually not encouraged.

The aim of mathematics education for most teachers is for students to learn procedures and attain good marks. But it is the understanding of deeper conceptual underpinnings which is important. If one develops insights here, these insights are likely to stay with one throughout life. Mathematics is then likely to become less burdensome as one starts seeing patterns and relationships behind events happening around us. And Channa was constantly chipping away at the deeper and mysterious structures of the mathematical world, inviting us to explore and savour their beauty along with him.

As I write this, I took easy recourse to the Internet to check if what I remember from Channa's classes was correct.

The World Wide Web (or 'www') is a place where everything we discussed in school with Channa is easily available – in print, videos and all sorts of forms. You can access all the fascinating stuff and it will keep you occupied for a lifetime. At the same time the Internet can dumb you down. It provides many things on a platter and you are tempted to copy-and-paste from the myriad notes and articles and call them your own! No wonder then that there is software developed to detect plagiarism.

But in the 1980s when Channa taught us, the 'www' was not even in our wildest fantasies of the future. Therefore he must have read a lot, sitting in libraries and looking for his favourite books in bookshops. When some of us got a glimpse of his private library, we knew he was a serious reader of mathematics, science and literature in general. His study was filled with hundreds of books, all arranged in haphazard but beautiful ways, wherever space was available. Perhaps that's where I developed a penchant for setting up my own library.

When I looked back on those three years that he taught us, I realised he was an avid explorer of the mathematical world. This, I now believe, is a prerequisite for every teacher of the subject. A teacher cannot limit his or her teaching to sharing the procedures to solve problems, but they must share the real excitement that comes from exploring patterns and relationships. And they must do this in a way that children can understand. Access to good reference materials can help, but the basic requirement is curiosity. This is a critical marker of a teacher. Can one convey excitement to the child without being curious oneself?

No wonder then, that Suresh Menon said of Channa:

> "We absorbed through some kind of osmosis that even we were not aware of."

The quest for truth – isn't that what education is supposed to be about? Yet we bombard children with facts and then we want them to regurgitate these facts in their examinations. We, the paranoid adults, perennially anxious about the rat race.

In mathematics, the quest for truth happens through an enquiry into the underlying patterns and relationships in space, with numbers and the like.

Channa kept peeling away the layers of truth, one by one. It had been like that right from the beginning. Along the way, we experienced the beauty of this process. Each step revealed this beauty, and each step illuminated the truth, little by little.

Channa wanted each and every one of us to join him in this quest. Ultimately, that's all there was to it.

9 His story

"A teacher who lets you think... and let's go... that is a rare species!"
– Guru Murthy (class of 1999)

The Channa I know, the Channa thousands of others know, would not have been the man he is today had he not been born in Mattur village, an interesting and picturesque place in Karnataka on the banks of the river Tunga. In this sleepy community, Channa and his friends swam in the Tunga and played in the lush green paddy fields dotted with areca nut trees. They also discussed maths.

Mattur is a little over 8 km from a town called Shivamogga, also known as 'Shimoga'. The town is the headquarters of a district of the same name. Channa was born in British India in 1937 and went to school in Shimoga. He was once offered a 'double promotion' to jump from the sixth to the eighth grade. He stood his ground and did not accept the promotion, despite much pressure from his teacher and his father. He simply felt that he had to clear one class at a time. His teacher and his father had to relent.

When he was 14, the family shifted back to Mattur since his father's business in Shimoga was not doing well. But his schooling continued up to the twelfth grade – known as the 'intermediate' in those days – in Shimoga. In the 1950s he moved to Tumkur, then a sleepy town not far from Bengaluru, where he completed his Bachelor of Science (BSc) degree in 1958. Following this, he began his first stint as a teacher of mathematics in Nirmala Girls' High School run by the Sisters of the Little Flower of Bethany in a place called Sagar, a little more than an hour's drive from Shimoga.

Channa charted out his own unique path as a teacher of mathematics from the beginning of his teaching life. His engagement with mathematics went beyond the routine preparation limited to the textbook and the syllabus. His avid explorations of the subject were soon reflected in his approach to teaching maths. He developed a razor-sharp clarity in the way he communicated the topic. At the same time, given his deeper understanding, he could recognise that there was no one sacrosanct method of solving a problem. There were often multiple approaches and each one was valid.

For a maths teacher, recognising this is important. You will discourage a child if you say, "This is the only or right way of doing it," especially when there can be different approaches to the problem. During his teaching stint at Sagar, he once recognised the originality of an approach used by one of his students to solve a problem in the geometry of circles. He immediately praised her unique approach and the correctness of her solution.

As Channa began his career in right earnest, a crucial aspect emerged that soon characterised his approach to the subject and its teaching. He developed an appetite for solving challenging problems. This passion for attacking a problem with gusto did not dim with age. The earliest example of this indomitable spirit is again from Sagar, in 1960. Our Channa was then a few years into the teaching profession, full of enthusiasm, as you can imagine.

During a school science exhibition, a student brought a geometry problem from the book *Fun with Mathematics* by

Jerome S Meyer. She wanted to pose this problem as a challenge at the exhibition. The problem, popularly known as the Steiner-Lehmus Theorem and dating back to the 1840s looks deceptively simple. The theorem remains popular to this day, with many different ways of proving it (at least 80, according to sources), making it pretty unusual for a theorem.

In his enthusiasm and confidence, Channa agreed to pose this challenge, not realising at that time that the proof of the theorem was extremely long, unlike any other proof that one studies at the high school level. In fact, the student who brought the problem to him had already checked with her friends in another school where the teachers had dismissed it, stating that it did not have enough information. But within a day, Channa had worked out the proof. He presented the problem in the exhibition with the following caption posed on the blackboard: “Whet thy wit!”

Channa embarked on a different path from the beginning, a path defined by intellectual exploration and an appetite for challenges.

The years passed quickly. In 1961, he married Sharadamma. She was quite unlike him but they got along well. In 1962, Channa moved to Bengaluru, did his BEd from the Mysore Education Society (MES) College and taught in the Rashtriya Vidyalaya (RV) School for a few months. Sharadamma later taught at the Kamalabai Girls’ High School, off Queen’s Road in the cantonment area, near my ancestral home at 14, Muniswamy Road. In January 1964, Channa joined the Baldwin Boys’ High School. At that time Baldwins had vacancies for teachers but usually tried to fill these with teachers from the Christian community. They were not always successful. That is how Channa, Sanna and Pasha were able to enter the institution in the 1960s.

Situated on the right bank of the Tunga River, Mattur is interesting because it is one of the few villages in India where many people speak Sanskrit. The population is mostly Brahmin, though there are other communities as well. Agriculture is their mainstay, and with plenty of water available paddy and areca nut are cultivated there.

Mattur still has a predominantly 'Vedic' culture, if I may use the word. Impressed by the level of Sanskrit scholarship there, the pontiff of the Pejawar Mutt (one of the Ashta Mathas belonging to the Dvaita school of philosophy, founded by Sri Madhvacharya), Visvesha Theertha, declared the village a Samskruta grama or Sanskrit village way back in 1982. Even today, there are Vedic pathshalas in the village, with scholars from outside India studying in them. The house in which Channa lived and grew up is a pathshala today. It may not be unusual to find men, women and children in Mattur conversing in Sanskrit, which is taught in the village school. Even the non-Brahmin populace uses Sanskrit for communication. I find this very interesting.

The Brahmins of Mattur call themselves 'Sanketi' and are ancestrally connected to 'Nacharamma', a Sanketi who migrated from the Tamil Nadu-Kerala border a few hundred years ago during the time of the Vijayanagara Empire. The Sanketis were the custodians of the temples and of religious learning. In fact Channa's father, Venkatadri Shastri, was an Advaita Vedanta 'vidwan' (a scholar of the Advaita Vedanta philosophy) with a postgraduate degree in Sanskrit from the Maharaja College of Mysore.

Channa's childhood days were spent in this Samskruta grama. He became proficient in Sanskrit at a young age, reading and writing it effortlessly. And he wasn't bad at conversing in it either. Ah! That explains the Bhāskara and *Lilavati* connection.

"There was this chap called Venkatram in our village who was an avid math student," Channa remembered. *"He was interested in astrology as well. He had a copy of Bhaskarāchārya's Lilavati in Sanskrit, which I started reading even before going to college."*

During his graduate days in Tumkur, Channa's teacher was Dr TS Subbaraya (TSS), who taught physics. He was also the principal of the college that Channa studied in. Another teacher, Dr TH Venkatasetty (THV), taught physical chemistry.

"Dr TSS was highly disciplined, a stickler for time. His lectures on physics were popular and I could make out that he was solidly grounded in the subject." Channa's respect for him grew as the days went by. And when TSS told his class how he went to great lengths to discover the truth, he created an exalted place as a teacher for himself in Channa's scheme of things.

In one of Dr TSS's public lectures at a College Day function, Channa heard a discussion about a problem from GR Noakes' book *Light*, which had appeared in an exam. This is how it went: There is a pond at the bottom of which lies some printed material. A swimmer dives into the pond carrying a lens with focal length 'x'. He wants to read the printed material. At what distance will the printed material be clearly magnified when he uses the lens to read it?

For some reason TSS suspected that the answer given in the book was wrong. He took some extraordinary steps to prove his point. Since he did not know swimming, he filled a bucket full of water and performed a reading experiment using a lens, in it. His solution (the right one) was different from the one Noakes had given in his book.

He wrote to Noakes about his experiment and the result he had obtained. Noakes recognised that he was wrong and promptly responded to TSS acknowledging his observations. As a mark of his appreciation he even sent 1 guinea (21 shillings in those days) to TSS, who had a difficult time converting the money into Indian rupees!

This was a powerful story for Channa, a story of a person's quest for the truth and his unwillingness to accept something that he strongly believed was wrong. The story has stayed with him to this day. In many ways a quest for the truth permeated his teaching throughout his life. One simply had to get to the bottom of the matter.

I began to see the connections between Channa the student and Channa the teacher. Surely these great teachers left their mark on young Channa way back in the 1950s and 1960s. And Venkatram, his older friend and neighbour, also played a role in awakening his interest in maths even before that, in the by-lanes of quaint Mattur. The thought of becoming a teacher must have played on his mind from a very young age even if nobody put it there.

Dr TSS was an 'early inspiration', the one teacher who left a deep imprint on his mind. What Channa liked about him was his humility. There were also the inspiring stories he often heard about Dr TSS's conceptual arguments with Sir CV Raman, in which he refused to be convinced by the Nobel Laureate's arguments if they were not clear and rigorous. *"The man had guts and loads of conviction about his own understanding of the subject. His arguments and debates with CV Raman are the stuff of folklore"*, said Channa.

So, it's quite likely that the physics professor strengthened Channa's resolve to become a teacher. The urge to find out about things, teach and share the excitement of discovery gripped him more with each passing day.

Dr THV was the one who introduced Channa to George Gamow's famous book *One, Two, Three… Infinity*, which set him on a trajectory of discovery and learning. He himself later strongly recommended this book to Suresh Menon's class in 1975.

So, things kept getting connected with one another as I discovered more of Channa's beginning years. I could tell that those early experiences were definitive. His childhood exposure to Bhāskara's *Lilavati* stayed with him and we were treated to problems in algebra and trigonometry year after year from the treatise. For Channa, reading as a hobby was something that he developed during his schooldays and it stayed with him thereafter. *"The teacher should essentially remain a student and reading helps him stay that way."* Reading helps you enter multiple worlds, including the mathematical.

On that first day of grade 8 in June 1982, Channa brought to us, with full force, his immense wisdom of mathematics and its teaching. What I didn't know at the time was how long that adventure would last.

The early encounters with Bhāskara's Lilavati and Bijaganita bestowed upon Channa the gift of mathematical combat, a gift that lasted several decades and whose influence gets only stronger, it seems.

There is a computer tucked away in a corner of his study. It is loaded with a software that allows him to communicate his mathematical solutions to tricky problems to journals of repute. Unlike the teacher who might be content just teaching from the textbook, Channa seeks problems to solve from diverse sources, for he believes that such problems offer newer insights into the structure and beauty of mathematics.

While the computer occupies a corner, the books are all over the place. They lie carelessly, here and there, as if knowledge is fluid and knows no boundaries. I'm about to pick up a book. He moves in to gingerly dust it before I can take it in my hands.

10 Infinity

It's enough. I did what I could... the thought (of continuing) kept lingering, though. But the old order changes... they have fixed an age for people to retire, not without reason. When it is time, one has to move on.'
– Channa

Channa and I are having a discussion at Vasuda, his residence at Banashankari in south Bengaluru. There are so many facets to this man that I often feel I do not measure up to the task of writing about him. Every interaction throws up something new. Given my work pressures over the past few months, I have not been consistent in my writing. But I want to finish this book as soon as I can.

I must tell you what Channa did after he moved on from Baldwins in 1999. There was this old student of his who was a trustee in another school in the neighbourhood - Bethany's High School. Channa was invited to teach there. But he stayed there only for a couple of years.

"How could they even allow you to leave Baldwins? Didn't they see that you would be invaluable as a guide and mentor to other teachers… especially the one's teaching maths?" My exasperation was apparent.

He saw where I was coming from and just smiled. Perhaps he wanted to share a few things, but chose not to, on second thoughts. This made me think about how educational institutions such as schools can draw on the wisdom and experiences of good teachers who have moved on. I'm not sure if this idea has been explored enough by our educational institutions. I can say that it hasn't been done so, at least in Channa's case. Sadly, the school did not think this was needed. The students were disappointed. They often asked Suveer, his grandson who was studying at that time in Baldwins, to bring him back.

Interestingly, Channa's wisdom was drawn upon by another very old institution in Baldwins' neighbourhood, the St Joseph's European School, which invited him for a lecture to inaugurate their maths club and seek his advice about its functioning.

"I have given my life to math teaching," he went on, as I kept ruminating. Which is why in June 1995, he was honoured in Bengaluru by the Rotary clubs of Mid-town and Peenya with The Governor's Award, in appreciation of his *"yeoman service rendered in the field of education"*. The award citation noted the high esteem in which his past students still hold him. The Rotarians also noted his other interests – debating, swimming, chess, photography and astrology. Some 21 years later, Channa's students again got together with the Rotary Club of Cubbon Park to honour their teacher.

The astrology connection is interesting and Channa pursues it actively to this day. The knowledge of numbers does open a door to the study of human behaviour, with astrologers even claiming that it can help predict life's outcomes. This claim is, of course, a debatable point, with rationalists rubbishing it. As Channa himself points out, *"I don't completely trust it. It only broadly indicates a person's natural inclinations, disposition, mental acuity, etc, and cannot be taken as a finality. But one could strive to overcome the shortcomings indicated by astrology."*

We come back to teaching. *"Teaching is pleasurable but stressful. Actually, teaching and evaluating go together and it is evaluation that is stressful. Imagine teaching without some form of evaluation, however limited it might be in its scope."*

I couldn't agree more. I have been a teacher too, and I can understand another teacher's language. Evaluation is finding out whether what has been taught has been learnt well, with some level of basic understanding. In Channa's time, this was done mainly through a myriad of tests and exams that we were subjected to month after month, year after year, in which we were given some numbers that were supposed to indicate how good or bad we were as students. Much as the examinations are a torture for students, evaluation is a stressful activity for teachers, especially when you need to do it for large classes and in schools that pride themselves on their results.

Nowadays, we have what is called 'Continuous and Comprehensive Evaluation' – CCE, in short – in which teachers are expected to evaluate not just at regular intervals but also in a more holistic manner, going beyond the subjects taught to include the so-called co-curricular areas as well. I'm not saying that the idea of CCE is bad, but the fact is that the poor teacher ends up mechanically filling so many forms, not knowing who will read them and use the information. Anyway, that's another story.

There was a final comment from Channa about moving on, *"I'm not bound and don't want to be bound."* It sounded like, *"That's it! I won't change my mind,"* which was really what he thought when he decided to hang up his boots – after more than 40 years of passionate teaching!

"Do you do any mathematics now?" I ask, curious to know if he continues to engage with the subject at the age of 79.

"Yes, I do."

He didn't say much. As we continued our discussion, I noticed a book on the table in the living room. Channa was savouring it, perhaps for the $(n+1)^{th}$ time. I picked it up and read the title on its yellow cover page: *Introduction to Analytical Number Theory* by Tom M Apostol. I flipped through the book and saw mathematical mumbo-jumbo on every page. Apostol was a passionate author and well-known analytic number theorist from the California Institute of Technology. In his book, he challenges you to enter the rewarding world of numbers to unravel their relationships. What looked like Greek and Latin to me was sheer poetry for Channa. Or so it seemed. He is among the few teachers who continue to live in a world different from that which is occupied by conventional teaching, a world he showed to thousands of us with gentleness, warmth and humour.

Clearly the man is still at work. He probably enjoys it even more now, even as he keeps attacking more and more problems and regularly submits their solutions to journals. No one else in his family reads books like the one by Apostol. His children and grandchildren have all branched off into other domains. Of course, they benefited from his maths and his wisdom. This, they tell me in our conversations. His grandson Suveer, who spent much of his growing years with Channa, echoed my own observations:

> "He made math interesting by telling us a lot of stories. Now kids like stories, right? You introduce any topic with a story and it becomes interesting."

I can imagine a poignant moment. I picture a young Channakeshava in the by-lanes of Sanskrit-speaking Mattur with Venkatram nearly 60 years ago, when the world was a very different place. I see Venkatram handing over Bhāskara's *Lilavati* and *Bijaganita* to Channa, challenging him to solve some problems in it. Such must have been the nature of that exchange between the two that Channa made it a point, year after year, to treat his students with problems from the *Lilavati*. No wonder they say that influences in the early years are important in every individual's life.

Apostol's book in Channa's living room brings back memories of my interest in theoretical physics and mathematics during my days as a student. I had been fascinated by these disciplines thanks also to my childhood hobby of astronomy. During my engineering days, I had bunked classes with a few maverick friends to ponder over fascinating areas of physics such as classical mechanics, relativity and quantum physics. Our quest had been to understand the nature of reality. I thought I would do nothing else, and end up becoming a theoretical physicist who would unravel the secrets of nature for everyone to see. There is something about physics which enables you to ask fundamental questions about the universe in which we live. That's what makes it so exciting.

But life had other plans, and so I ended up becoming a teacher. My friends entered reputed research institutions to pursue their dreams of discovering nature's secrets through maths and physics. My inclination was more social. I felt our insipid educational system, both in school and at the higher level, prevented children from embarking on the fascinating journey that is learning. That was the problem which needed to be addressed. So I set out on the path of teaching children. I wanted to share with them the joys of learning, of discovery, of developing insights into the nature of things. I believed passionately that one could change the world this way. That belief has never left me.

As my engagement with education and its problems grew over the years, especially with schools run by the government, I began to realise that schools are located within a larger society that is complex and that has its own ways of ranking, sorting and filtering children. Not everyone ends up with the same educational opportunities. There are no simple answers to these social problems because education is a 'contested' terrain, with multiple views on what it should be doing for children. There are many different opinions about why children should be educated, what they should learn, how they should learn and so on. There are also many different theories about learning that have spawned a range of teaching approaches. Each approach is keenly debated and pushed by its proponents. That's what makes education so fascinating.

My tryst with education has led me to delve deeper into the nature of human society, in particular our Indian society and its unique and persisting problems of caste, class, religion, culture and politics – and how these factors influence what children ultimately learn (or not learn) in schools. To cut a long story short, my concerns over the last 20 years or so have essentially been social. Yet, in this din, I have not forgotten my earlier affair with mathematics.

I realise what a joy it is to understand both worlds. Mathematics shows the way to understand the workings of the physical universe, while the task of educating children opens you up to questions about the social world, what it is like now, why it is the way it is, and what we can do to it through education. The idea is to lift the veil of my own ignorance by finding answers to these questions. The idea is to lead a meaningful existence as grand old Socrates is said to have proclaimed one day: "The unexamined life is not worth living." This piece of Socratic wisdom is at the heart of what we do in education. That's perhaps the reason why I was attracted to it in the first place.

The possibility of understanding different facets of our world by studying different areas of knowledge excites me. Writing about Channa helps me in my quest. In my lifetime I want to understand many mathematical truths and soak in their beauty. One of these truths is Kurt Gödel's theorem, which says that mathematics itself is 'incomplete' and can never be complete. Do you get that?! It's called the 'Incompleteness Theorem', propounded by Gödel in 1931. He was only 24 years old then.

"Mathematics as a system of rational thought is incomplete," he proclaimed. "There will come a stage when it will not be able to prove nor disprove some statements. This will place serious limits on how much we can actually know." Isn't this shocking, the way the term 'incomplete' is used to describe mathematics? This is a different kind of truth altogether and I would like to get to the bottom of it some time. I plan to drop by Vasuda again one of these days to begin this journey.

What is special about Channa is that he has always been a learner of mathematics. He has engaged with it seriously for many years, since the time he laid his hands on the *Lilavati*. This goes far beyond the preparation he would normally need to teach high school students. Almost as if reading my mind, Suveer added:

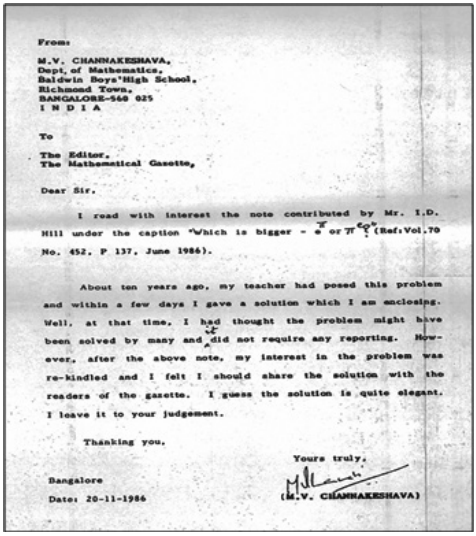
> "He has far more in him than his students ever managed to see or get. This I discovered in my high school years when he taught me vectors, cubic equations, calculus and other topics, much before I studied them in college. The man had a vast repertoire."

Let me share with you something I discovered as I came to know him better and better. For many years, from the 1980s onwards, Channa has been sending his contributions to the 'Problem Corner' of *The Mathematical Gazette*, a journal published in the UK. The gazette is old, in fact very old, having been established in 1894! It is meant for those with an interest in, and enthusiasm for, mathematics. The unique part is that the gazette connects schoolteachers, college and university professors, and just about anybody with an interest in the topic. It has been acknowledged as one of the leading journals in the field of mathematical teaching and learning. Channa has had an interesting relationship with the gazette. He told me a little story.

> *"My relationship with The Mathematical Gazette was by accident and not intentional! You see, I was a member of the British Council Library those days. An issue of the gazette was on a table opposite me when I once visited the library."*

"And then?"

"While I was turning the pages, I was struck by an article that posed the question: 'Which is greater, e^{π} or π^{e}?' I had worked this out some years ago. You see, it's not such a great or serious problem. But I thought I should send my solution to the editor of the gazette, which I did. The editor acknowledged it and it appeared in the next issue along with solutions sent by some others. From then on, I used to send solutions to the problems posed in the gazette whenever I got hold of a copy."



From:

M.V. CHANNAKESHAVA,
Dept. of Mathematics,
Baldwin Boys' High School,
Richmond Town,
BANGALORE-560 025
I N D I A

To

The Editor,
The Mathematical Gazette,

Dear Sir,

I read with interest the note contributed by Mr. I.D. Hill under the caption "Which is bigger - e^{π} or π^{e}?" (Ref: Vol.70 No. 452, P 137, June 1986).

About ten years ago, my teacher had posed this problem and within a few days I gave a solution which I am enclosing. Well, at that time, I had thought the problem might have been solved by many and it did not require any reporting. However, after the above note, my interest in the problem was re-kindled and I felt I should share the solution with the readers of the gazette. I guess the solution is quite elegant. I leave it to your judgement.

Thanking you,

Yours truly,

(M.V. CHANNAKESHAVA)

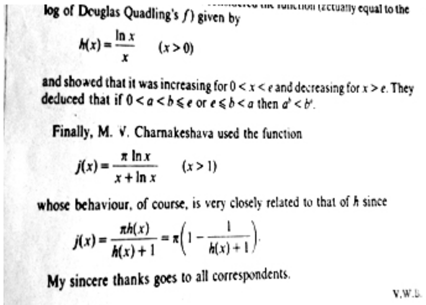
Bangalore

Date: 20-11-1986

Channa's letter to the editor of The Mathematical Gazette in 1986

Channa had told me, *"That problem of π is not very difficult – it is just a bit of calculus!"* But it has enough in it to flummox you. If we ask which is greater, 23 or 32, the answer is simple. You get into difficulties if you have to find out which of these is bigger: $2.5^{3.5}$or $3.5^{2.5}$? Since both e and π are irrational numbers, comparing eπ and πe turns out to be more difficult than you first thought.

Channa used some 'elementary calculus' to create a 'mathematical function' to show that $e^{\pi} > \pi^{e}$. The frayed copy of his letter to the editor of the *Gazette* is reproduced above. Note that the exponents had to be handwritten, as it must have been difficult to type them in those days. There is also the portion from the journal that acknowledges Channa's contribution, given below. There are many such examples illustrating Channa's deeper intellectual engagement.



... the function (actually equal to the log of Douglas Quadling's f) given by

$$h(x) = \frac{\ln x}{x} \qquad (x > 0)$$

and showed that it was increasing for $0 < x < e$ and decreasing for $x > e$. They deduced that if $0 < a < b \leqslant e$ or $e \leqslant b < a$ then $a^b < b^a$.

Finally, M. V. Charnakeshava used the function

$$j(x) = \frac{\pi \ln x}{x + \ln x} \qquad (x > 1)$$

whose behaviour, of course, is very closely related to that of h since

$$j(x) = \frac{\pi h(x)}{h(x) + 1} = \pi\left(1 - \frac{1}{h(x) + 1}\right).$$

My sincere thanks goes to all correspondents.

V.W.B.

The UK journal's acknowledgement of Channa's contribution

I have chosen to discuss this to show how a teacher of school-going children has dabbled in his discipline at a much deeper level, more like a mathematician. You may ask if teachers need to do this. I would just say that teachers should not only pass on facts or information or instructions or, in the case of teaching mathematics, procedures to solve problems. A deeper engagement with the discipline would greatly help in their growth.

This is the great tragedy of our times – we mostly want our children to 'get the answers right' and somehow crack the exams. One of the key purposes of teaching maths is to get children to think mathematically – to appreciate patterns and relationships, to ask questions, and generally, to be curious. This is what Channa tried to get us all to do. For teachers to teach in this way, they have to engage with the subject in many different ways. With some sustained support, they can.

The sad part is that we do not prepare teachers to be eternal learners. Our training does not make teachers explore their subject of interest and ask questions. Teacher training needs nothing short of an overhaul.

Channa engages constantly with mathematics, as a student. This is what keeps his mind active many years after he has stopped teaching at Baldwins. His interest, I surmise, has helped him on three counts – one, it did not let him slip into a mundane existence as a maths teacher, teaching the same formulae and exam tricks year after year; two, it made his teaching very rich; and three, it enriched his life and the lives of his students as well. That is what happens when you seek truth and meaning.

“Can we put all of what you did in the curriculum? Should we?” I’m interested to know what he thinks. My argument at that point is that if it appears in the curriculum, it must then be taught. How nice it would be, I think, if we can ask the teacher to discuss paradoxes and famous problems in mathematics and use this to introduce us to various concepts and fire our imagination, like Channa did.

“Oh! That would be quite disastrous, I think. You can’t make these things compulsory. Refresher courses may help. One can also develop reading materials for teachers. But ultimately, one has to leave it to them.”

There is no prescribed formula.

I agree. Developing texts for teachers is important. They need greater and easier access to knowledge. One may cynically ask, “How many teachers read nowadays? Or care to?” But can we focus on those teachers who care, who want to walk the talk, and who want to do something different? The point Channa makes is that teachers should keep learning anew, even the experienced ones. Otherwise, teaching can become a boring and repetitive routine.

The challenges of teacher preparation do not have easy answers, especially the bit about motivation. If we can crack that one, there would be a radical shift in the way teachers teach and children learn.

I wonder what it would have been like had Channa taught in an institution of higher education. Perhaps that is what he had in mind when he first left Baldwins in 1976 after easily managing to clear the exam to enter the prestigious Indian Institute of

Technology (IIT) in Madras. There Channa pursued a postgraduate degree in the subject he loved. This was an intellectually enriching period of his life, as he got an opportunity to explore his passion in a professional setting with those who actually engaged with the discipline. His wife held fort during that time managing the house when his earnings dipped.

But fate had other plans. Though Channa completed the course and appeared for the final semester exam, the cruel irony was that the institute subsequently wrote to him saying the answer scripts of the entire class had been lost, and hence could not be evaluated! He had to rewrite the entire exam at a few days' notice. Given very little time for preparation, he could not complete the exam this time round. And then there was the difficult decision of coming back to Bengaluru, back as a maths teacher at Baldwins, letting a promising opportunity slip by…

"He could have continued at IIT if he had wanted to." I'm discussing Channa with his elder daughter Rohini in the living room. His face is hidden behind a newspaper and I have this feeling that he is silently listening to our conversation. "He could have gone on to teach at college. But he was looking for stability. He had a family to run. His brother and sisters were dependent on him." Channa was the go-to guy for his siblings, right from the beginning.

Suveer interjected, "Channa does not stick to the past. I have never seen him brood over things." He resolutely carried on, never for a moment giving up on his love for maths. The explorations continued with all the responsibilities he had of managing his family.

Ramachandra, Channa's son-in-law, who was quietly watching this conversation unfold, asked pointedly, "Who knows what may have happened had he continued at IIT. We all know about Ramanujan and what he contributed as a result of his collaboration with Hardy." I nodded. Who knows…? A nurturing environment can indeed have a magical effect.

There is a heaviness in the air. The question uppermost in my mind is, “What if…?” I would never have met Channa then. The four or five thousand children whom he taught over four decades would not have met him either. This book would never have happened. But the great thing about this man is that he did not let his disappointment and bitterness over the IIT episode overcome him. His love for mathematics transcended these difficult happenings. He came back and carried on undeterred with his exciting explorations of the subject he loved. And we gained from his stories and insights.

On another note, I would like to make the point that while institutional support, an enabling environment and professional affiliations are important, nothing can stop a genuine individual from the pursuit of knowledge. Channa has exemplified this in the best possible manner even during trying times, during times when it was far more difficult to access knowledge. Today, all it needs is a click of a button. But digital technology creates its own problems, which I do not want to go into here.

We move on even as I reflect on these possibilities. I am curious to know what Channa reads apart from maths and science. He reels out a long list of authors whom he has read over the years, in languages as diverse as Sanskrit, Kannada and English, and on themes that touch virtually upon all aspects of human existence. Bankim Chandra – the writer and poet from Bengal; Shivaram Karanth – the Kannada writer, activist, thinker and environmentalist, described sometimes as the ‘Tagore of modern India’; SL Bhyrappa – the Kannada novelist whose writings focus on the ‘search for truth’; DR Bendre – a most gifted Kannada poet; PT Narasimhachar – Kannada playwright and poet; Maxim Gorky – the Russian writer; Maupassant – the French writer and foremost exponent of the short story; Somerset Maugham – the British playwright and novelist. The list is long.

“What have you got from these writers?” I expect a lengthy articulation in response, and am prepared for some furious scribbling in my notebook. But Channa just says, *"It broadens you as a human being."* After some prodding, he adds, *"All these writers explore human values intensely… and I'm attracted to the idea of eternal human values that are immutable."* That's it, I suppose. Ultimately everything boils down to seeking the truth as one sees it. And that is how consciousness widens and deepens, allowing one to become a fuller human being.

"I suppose his thirst for knowledge actually came from his father," Rohini had chipped in. "Other than math, which he has never let go of, he kept learning many things, like music."

Learning is enjoying. There is no question of Channa hanging up his boots.

Channa's father, I was told, was a highly inquisitive person. Even when the Vedic scholar was laid low by a stroke and glaucoma at the age of 80, he would continue to engage in long conversations with anyone who talked to him. "Hey, come here," Venkatadri Shastri would often say. "Let me also learn." His demise affected Channa deeply. He went into a period of quiet introspection.

"At 60, my grandad taught himself C programming," Suveer said, "when he realised that everyone was beginning to use computers to solve mathematical problems. To top it, he enrolled in a distance course from The Indira Gandhi National Open University to do his Masters in Computer Applications." Channa did not go on to complete it, though.

The most striking observation which comes from my conversations with his family is Channa's steadfastness in protecting his values. This was far more important to him than making money.

The conversation keeps meandering, and I suddenly remember to ask him about Sanaulla, the rotund Hindi teacher we were so scared of. Channa and Sanna got along well and we could make out that they were great buddies. They would, in today's parlance, 'hang out' often and laugh heartily together.

After much effort, I finally land up at Sanaulla's doorstep in Koramangala. Channa has given me his number. Sanna has just returned from his Haj pilgrimage, and is delighted to know about the book. He is very warm. We begin talking in his living room:

> *"We three – Channa, Pasha and I, had similar likes and dislikes... non-smokers, non-drinkers. We were inseparable – the three musketeers on their iron horses."*

Channa's younger daughter Ranjini had another name to describe the three musketeers. "We used to call them Amar, Akbar, Anthony." This is the title of a famous Hindi action-comedy film from the 1970s, which is about three brothers separated in childhood and raised in three different faiths – Hinduism, Islam and Christianity.

The information about Pasha being part of the gang was news to me. It didn't seem obvious to us as students. Sanna went on, his eyes glazing a bit:

"Channa is selfless and contented... and he is greatly knowledgeable. He is very gentle, sincere and down-to-earth. Above all, he is an excellent teacher and a very good human being. We have been very close even after leaving school."

When Pasha passed on, Channa and Sanna were with his family to share their grief. Channa had then said, "When the time comes to pass on, one must just go without any fuss… no need to get admitted to the hospital and trouble everyone." Sanna couldn't agree more. Tears welled up in his eyes as that statement came back to him.

At Vasuda, as another day dawns, Channa continues to furiously work away, looking infinity in the face.

Epilogue

"Have you ever really had a teacher? One who saw you as a raw but precious thing, a jewel that, with wisdom, could be polished to a proud shine? If you are lucky to find your way to such teachers, you will always find your way back."

- Mitch Albom (In Tuesdays with Morrie)

December 5, 2014. Nearly 30 years after I graduated from Baldwins, I'm again on my way to Vasuda, Channa's residence. It is my big occasion and I'm wondering what I should say. As the taxi speeds towards Channa's house, I am tongue-tied and a bit restless.

'Yesterday, Once More' is the catchy title of an Old Students' Reunion at a well-known hotel in the city. A few hundred students are expected at this annual bash, where there is a lot of noisy music, dancing, catching up, loud laughter and booze. I had never been to such gatherings, cut off as I had been from school and school-friends after 1985. But this reunion is to be a bit different. It will begin by honouring Mattur Venkatadri Channakeshava, the gentle man who taught us infinity. The Baldwin Alumni Association readily agreed when I made a request that we use the occasion to honour Channa by gifting him the book that I had written about him.

At 6 pm, I knock on the front door of Vasuda. I am ushered into the living room. Channa is ready and waiting. He looks immaculate in his trademark brownish suit. It brings back memories of another day. "I had trouble fitting into the pant," he says, laughing. "Didn't realise I have put on some weight around the middle!"

We then go to his study, where he excitedly shares with me his latest contribution published in the 'Puzzle Corner' of the

Australian Mathematical Society's Gazette. It is an interesting problem that challenges your ability to geometrically visualise what happens when a $1 coin (shaded in the image below), which touches three other $1 coins lying on a flat surface (as shown in Figure E1), rolls around them, touching them the entire time, until it returns to its original position. Question: How many times does the coin rotate relative to its centre? *The Gazette* published Channa's solution. I feel so proud to see it in print.

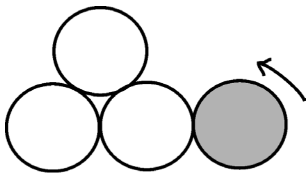


Figure E1

Doing maths is his lifeline. Living, for him, is learning, constantly expanding the boundaries of his understanding.

In 45 minutes, we are at the Windsor Manor, at the entrance to the hall that is the venue of the reunion. From the moment we enter, Channa is kept busy. His students come up to him. Some shake his hands and welcome him. Others touch his feet even as they introduce themselves. Out of the corner of my eye, I can make out that Channa has not forgotten most of them.

In that din, Supreet, a classmate of mine who passes by, tells him, "Sir, I have still not forgotten QED!" Whoever comes up to him recalls his favourite Channa anecdote.

Mrs Thomas appears, almost as if from nowhere. Yes, Mrs Thomas, our kind and ever-smiling class teacher who taught us maths in class 6 in 1980, some 34 years ago. Someone very thoughtful has arranged for her to come. For a moment, I'm speechless. I then go up to her, introduce myself and tell her about the book, adding that she is mentioned in it. Quickly I get a photo of us taken together. Later, I manage to read to her the paragraph in the book that mentions her. She is too surprised for words.

The function then starts.

After the customary introduction, we, the old students, are asked to sing our school song – the Baldwin Girls' school song first and then ours. Here we go:

"Shout 'All hail' for Baldwin High School

Shout it loud and long

Our beloved Alma Mater

Come and join her song.

Lift the banner, bear it onwards,

Righteousness and truth."

As I sing with fervour, I remember my schooldays. Among those myriad images and memories that come rushing by, I remember my tryst with Channa. He taught us to explore truth through maths. He taught us to see beauty in it. Tears well up in my eyes.

And then, as the group settles, I'm invited to speak. I request Channa to join me on stage. Still at a loss for words, I begin by saying that this is a very emotional moment for me, nearly 30 years after I passed out of school. I go on to explain the idea behind the book, about the need to honour a great teacher. They all listen.

The big moment arrives. I have neatly gift-wrapped the volume. I hand it over to him, thanking him for being a great maths teacher. He struggles to remove the wrapper, needing my help. We hear the applause. I feel relieved and relaxed all of a sudden. It's all over. Or maybe, it has all just begun.

Channa is then invited on stage to share his thoughts. He does not say much, as is his wont. But he lets his modesty show, "I kept asking Giri if this is important, if it will help… I hope it does."

I think, "That is for the world to decide, Mr Channakeshava… don't you worry about it."

Additional notes

"There is no royal road to geometry."

- Statement attributed to Euclid, circa third century BCE

First things first – don't get scared. As I have tried to show throughout this book, maths should not be learnt or appreciated in the manner followed by most of our schools. Yet, as Euclid is supposed to have said to an agitated King Ptolemy I who ruled Egypt more than 2,000 years ago and was finding it difficult to learn some geometry, there are no shortcuts. But if one is prepared to pay close attention to the subject for some time, the experience of engaging with maths can be uplifting. Now that you have come thus far, I will try to further enliven your journey and make it more exciting. So there is more stuff for you to munch on. I hope this will help turn you into a willing explorer of that wonderful world we glimpsed with Channa. The additional stuff is related and limited to what has been discussed in the book. I have segregated it chapter-wise.

CHAPTER 1

1.1 The numbers e, π and i, which appear in the Euler equation

We discussed the Euler equation $e^{i\pi} + 1 = 0$ as an example of great beauty in mathematics. Richard Feynman and others have said very interesting things about this equation.

But what are these numbers e, π and i all about? Even a 'basic' appreciation of them can help us understand why several reflective statements have been made about this equation. I can only attempt to generate a basic understanding of the equation and an appreciation for its elegance, and hope to motivate you to explore further.

Let me begin by saying that e and π are called 'irrational numbers'. Simply put, a number that has a decimal/fractional portion which never ends and can never be computed is an irrational number. Also, the decimal portion does not have any particular pattern that we can discern. Imagine numbers like that! No wonder they are called irrational. Both e and π cannot be expressed as fractions in the form p/q, where p and q are integers. On the other hand, 'rational numbers' can be expressed in the form of p/q – for example, 1/5, 1/7 and so on. Note that these fractions do not generate the endless decimal portions that e and π do.

"What about the value 22/7 for π?" you may ask. "Isn't this in the form p/q?" But 22/7 is only a neat approximation of π, which we use while solving high school problems in geometry. It yields 3.14, correct to only the second decimal place. So the next time someone asks you what the exact value of π is, you should say you do not know instead of confidently dishing out 22/7 as the definitive answer. The ratio of circumference to diameter of a circle simply cannot be expressed in the form p/q.

The numbers e and π also have another name: they are called 'transcendental numbers', because they are not the roots of equations such as $x^2 - 2 = 0$, from which we get $x = \sqrt{2}$. In this sense, both e and π are different from $\sqrt{2}$, though all of them are irrational. I like the term transcendental because it indicates a sublime state. I guess e and π reside in that sublime state.

Let's begin with e, which looks like this: 2.7182818284… The three dots at the end mean there is no end to this decimal or fractional part of the number. This has been proved, but we will not go into that here. There are different ways of arriving at e. What interests me is that e has a long history going all the way to the ancient human activity of lending and borrowing. Lending became possible when humans started accumulating wealth, which coincides with the beginning of agriculture and the ownership of property. Motivated by greed, humans began to ask, "How can I create more wealth from my existing wealth?" The notion of earning 'interest' on a sum of money was thus born.

We learn about 'simple interest' in high school. It is simply a percentage of the amount (the 'principal') that is given as a loan. This is how lending or borrowing with interest began. If I loan you Rs. 100 (or 'P', the principal) and demand that you return the amount with 100% interest (or 'I'), the interest you will have to give me is Rs. 100 + (100% of Rs. 100). 100% of Rs. 100 is [(100/100) x 100] = 100. So you will return Rs. 100 (P) + Rs. 100 (I) = Rs. 200. Good going! I get double the amount after a year, like any greedy moneylender would. We now have a simple formula:

Interest (I) = Principal (P) x Rate of interest (R) = PR/100, since the rate is always a fraction with the denominator 100.

This arrangement sufficed for a few thousand years till a clever person asked, “Can I get more? This simple interest method isn’t giving me enough.”

One way is to increase the rate of interest. But this may not always be feasible and there could be vehement – even violent – protests by borrowers. Justifiably so. To get around this problem, someone had a brainwave many, many years ago.

He, or she, thought, “Instead of collecting interest just once – and that too at the end of a year or two – why not collect it more often, in fact, as many times as we can, while keeping the overall rate the same?”

This means we break the 100% interest into smaller parts, which allows the lender to collect interest as many times as he wants.

The greedy lender also hit upon another killer condition, “Every time the borrower returns the amount plus interest, he is given a new amount as loan that is equal to the amount lent earlier + interest paid on that amount. This process continues till the end of the loan period when the borrower finishes repaying as per the interest agreed.”

I know this is getting a little tedious, so let us take some numbers and see how this actually works out. It will be easier to visualise the process then.

Let me begin by lending you Rs. 100 at an interest rate of 100% per year. I know this is a ridiculously high rate of interest these days, but let us go with it for the sake of keeping the calculations simple. In the simple interest model, you will return Rs. 200 after a year, as we saw earlier.

But in this new model, instead of waiting for a year I say, “Please repay me every six months. I’m kind enough not to increase the interest rate for the entire year beyond 100%. Since there are two six-month periods in a year, I expect you to repay me two times with 50% interest each time." Got it?

So, after the first six months you return, without suspecting anything fishy:

Rs. 100 + (Interest (I) on Rs. 100)

This is nothing but 100 + PR (remember our simple interest formula where I = PR)

This becomes (100 + 100 x 50%) = 100 + 100 x 50/100 = 100 + 100 x 1/2

We can write it as 100 (1 + ½) = 150.

So this Rs. 150 is the principal plus 50% interest. But the entire loan deal isn't done yet. And here's where the catch comes in.

For the second six months, I make Rs. 150 the new principal, while the interest rate remains 50%. Remember, I said the overall interest rate would be 100%, which I did not increase. As per our deal, you are committed to return the money with 100% interest. Nothing less will satisfy me.

Thus at the end of the year, you must return: 150 (1 + ½) = 225 or Rs. 225.

We can do this calculation for the whole year in a single step. Since you paid the same rate of interest (50%) twice in the year, the final amount you owe me is $100\ (1 + ½)^2$ = Rs. 225.

Hmm… I'm on to a good thing, right? This is surely better than getting back Rs. 200 at the end of one year, with 100% interest being paid just once.

Now let's see what happens if I ask you to pay me back with interest four times per year instead of twice. You now pay 25% interest each time. But your principal keeps increasing each time as I add the interest to it. Using the same formula, we get $100\ (1+1/4)^4 = 100\ (1.25)^4 = 244.14$. So, I get Rs. 244.14 at the end of the year, which yields Rs. 44.14 of interest on the amount I loaned. This is getting us somewhere, isn't it?

I then become even greedier. Now I want interest to be paid every month, which is 12 times a year, which means 100/12 = 8.33% each time. So we get: $100\ (1+1/12)^{12} = 100\ (1.083333)^{12} = 261$. Wow! I'm extracting a full Rs. 161 in interest! Letting greed take over totally, I now order you to pay me with interest DAILY! I now get: Rs. $100\ (1+1/365)^{365}$ = Rs. 271.5 at the end of the year.

But there's something interesting happening here. Though the final amount I get keeps increasing as I increase the frequency of interest payments, it does not increase out of bounds but seems to settle somewhere. Let's stretch this a bit more. If I demand and get interest EVERY HOUR, then I end up getting Rs. 271.80. We now have enough data to create a simple table like the one below. The last column (a ratio) is nothing but the final amount divided by the principal. So, 200/100 = 2.0, for example.

Principal	Interest periodicity	Final amount	Ratio
100	1 time per year	200	2.0
100	2 times per year	225	2.25
100	4 times per year	244	2.44
100	12 times per year	261	2.61
100	365 times per year	271.5	2.715
100	760 times per year	271.8	2.718

If you look carefully at the numbers in the last column, doesn't it seem that we are tending towards our irrational e? No matter how much I increase the frequency (periodicity) of interest payments ('compounding periods', in standard parlance, which gives rise to the term 'compound interest'), I can never get more than Rs. 271.8281828459045… In ratio to 100, this would be 2.718281828459045…, which is our irrational number e. In other words, even if I increase the compounding periods infinitely, I cannot get more than e times the original amount. Phew! There we are. Let it truly sink in, this arrival at the abode of the mysterious e.

I must discuss one more point. We saw different expressions that multiply our original amount of Rs. 100 to give us our final amount: $(1 + ½)^2$, $(1 + ¼)^4$… $(1+1/12)^{12}$. Generalising, we can now look at $100\ (1 + 1/n)^n$, where n is the number of compounding periods, which one can increase as one pleases. So n = 1, 2, 3… Whatever this number might be, $(1 + 1/n)^n$ actually converges to e as n becomes larger and larger. Isn't that quite amazing…?

So this number e could have easily come out of an age-old human activity. Any phenomenon in the real world that increases in this fashion is said to increase 'exponentially'. The increase in human or animal populations can be exponential. There can be exponential decay as well. But mathematical imagination is not always bound by happenings in the physical world. So we can visualise e in terms of what is called an 'infinite series' in the following elegant manner (there are many ways of using infinite series to arrive at irrational numbers such as e and π):

$$e = 1 + 1/1 + 1/1.2 + 1/1.2.3 + 1/1.2.3.4 + \ldots$$

Isn't this interesting? Note that 1.2 here means 1 x 2, 1.2.3 means 1 x 2 x 3 and so on. So, there you are; e can be thought of in different ways.

I'm tempted to talk more about the idea of infinite series, a very exciting and useful topic in modern mathematics. But I shall leave it here. However, we will cite another example of infinite series when we discuss π in the next section.

1.2 The abode of π

Now, let's get to the other irrational, π. If there is any number with a long and chequered history that has caused sleepless nights for many a mathematician, it is this one. The definition of π is deceptively simple:

π = circumference (c)/diameter (d) of a circle, that is, the ratio of the circumference to the diameter of a circle, as every student in high school knows. This expression can also be written as $c = \pi d$ or $c = 2\pi r$, since diameter = twice the radius of the circle, which is usually denoted by 'r'.

To get a rough sense of this ratio, draw a circle, then cut a string that is as long as the diameter of the circle. Place this string along the circumference (path) of the circle. Now, keep flipping it over till you cover the entire circumference of the circle. You will find that the string will have to run a little bit more than three times, or 3.14 times to be a little more precise, before it fully covers the length of the circumference once.

One may very well think that this ratio of the circumference of the circle to its diameter would yield a decent, rational number that we can easily understand. That is, a whole number or at best, a number with a terminating decimal that shows a clear relationship between the circumference and the diameter of a circle. It is not difficult to believe that these parts of the circle are related to each other in definite ways. But when people from different cultures across the world tried to find this ratio centuries ago, they found it was a little more than 3. So they approximated the ratio to 3. But there was always that little portion beyond 3 that kept nagging everyone.

As the years progressed, people discovered that this little portion is a never-ending stream of decimal digits. In fact, it was proved beyond doubt that this ratio, when expressed in decimal terms, had the decimal digits running infinitely! So π, as this ratio was called, looked something like 3.14159265… without an end. You can draw a circle and find its diameter. But you can never accurately measure the ratio of its circumference to its diameter. That is, you can never know what the exact value of this ratio is. It was a ratio that stumped the great Pythagoras and quite shattered him – the realisation that there could be numbers whose actual values we might never know.

There is another similar example. We can draw a right-angled triangle whose sides are one unit each. We know from the Pythagoras Theorem that the length of the hypotenuse (the side opposite the 90^0 angle) is $\sqrt{2}$ (square root of 2), which is an irrational number. So, there you are – you can draw this triangle (which means that you can geometrically depict it) but when you try to measure the length of its hypotenuse, you will not get an integral value for its length. It cannot get crazier than this!

Given the exalted history of π, I would like to add a word about the heroic attempts to measure the number, including methods that used geometric approaches. An interesting and brilliant example is that of Archimedes (circa third century BCE), one of the greatest mathematicians of antiquity.

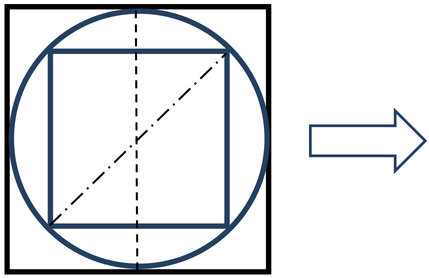


Figure A1

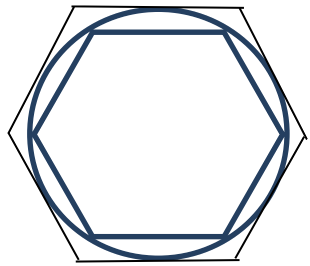


Figure A2

Archimedes reasoned that an accurate measurement of the elusive π depended critically on how well one measured the circumference of a circle. So he first drew a circle and inscribed a square inside it. Then he drew a square outside the circle (circumscribed the circle with a square). This is shown in Figure A1. The circumference of the circle is smaller than the perimeter of the large square and larger than the perimeter of the small square. For purposes of this discussion (to make our calculations easier), let us keep 'd' as the diameter of the circle (shown by the line with small dashes) and let 'd' be 1 unit. So 4d is the perimeter of the larger square. Now let the length of a side of the smaller square be 's'. So 4s is the perimeter of the smaller square. The circumference of the circle must lie between 4d ('P', upper limit) and 4s ('p', lower limit). Since d = 1 unit, the perimeter of the larger square is 4 units.

Now, 'd' happens to be the diagonal of the smaller square (shown in Figure A1 by the line with big dashes and dots in between). It is also the hypotenuse of the right-angled triangle with side 's'. Using the Pythagoras Theorem, we can express 's' in terms of 'd': $s^2 + s^2 = d^2$ or, $2s^2 = d^2$. Therefore, $s^2 = d^2/2$, hence $s = \sqrt{(d^2/2)}$. Since the perimeter of smaller square is 4s, we can say $p = 4 \times \sqrt{(d^2/2)}$. Since d = 1 unit, $p = 4/(\sqrt{1^2}/\sqrt{2}) = 4 \times 1/\sqrt{2} = 2\sqrt{2} = 2.83$ units. We can now confidently say that the value of π must lie between 2.83 and 4.00, given that the diameter of the circle is 1 unit, as we had earlier said.

"Big deal. That's not saying much," you might say. "The range is too large. Surely we can do better." So I reply, "Wait! Look at Figure A2 where we have trapped the circle between two regular hexagons." Regular hexagons have all six sides equal.

Here is a small exercise for you. Use and extend the method we have just used to find the upper and lower limits of π. The limits we get in the case of Figure A2 are between 3.464 and 3.00. Isn't this getting better? More than 2,000 years ago, Archimedes is said to have managed to trap the circle between two 96-sided polygons! I wonder how he did that with the rudimentary instruments he must have had at his disposal. Yet, he kept at it doggedly, and managed to trap the value of π between 3.141 and 3.143. Wow! By the sixteenth or seventeenth centuries, trapping π between many-sided polygons (often running into hundreds of sides!) took on the form of an obsession, leading to more and more accurate values of the upper and lower limits of π.

As the years passed, the struggle to rein in π acquired gargantuan proportions. With each attempt, mathematicians looked at π with greater respect and awe. It assumed an Everest-like quality in the world of mathematics. With the passage of time, as more powerful and analytical approaches began to be used, mathematicians discovered connections between π and other numbers. They slowly moved away from the geometrical methods to determine π. Like in the case of e, π was calculated with increasing accuracy by using the infinite series. I'm leaving you with just one fascinating, almost unbelievable example – the infinite sum of the reciprocal of squares, which converges to a value defined by π:

$$1/1^2 + 1/2^2 + 1/3^2 + 1/4^2 + 1/5^2 + \ldots = \pi^2/6$$

How on earth is this possible? We started off with π as the ratio of the circumference of a circle to its diameter. How did it get into a formula that does not have anything to do with a circle? What relation could a ratio, which has got something to do with circles, have with the sum of reciprocals of squares? Mathematics is truly intriguing because of all these unexpected connections.

1.3 Finally, *i*

Let us now take on *i*, which is what gives the Euler equation its special character. We need a quick recap of the idea of 'square roots' for this. No, square roots are not some fancy biological entities from the plant kingdom. Simply put, the square root of a number is a number whose square gives the original number. I know this sounds like a bit of a riddle. So let's take an example: square root of 16, or $\sqrt{16} = 4$, because $4 \times 4 = 16$. We can also write this as $4^2 = 16$. Note the symbol used to denote the operation of the square root. Interestingly, the square root of a number can be negative as well. So, -4 also fits the bill because $-4 \times -4 = +16$, with minus times minus becoming plus, as we learnt in school. Let me not get into why $(-) \times (-) = (+)$.

Can negative numbers have square roots? That is the moot question here. What would $\sqrt{-1}$ be? This is like asking: "What number when squared (multiplied by itself) gives -1?" This square root cannot be +1 or -1 because $+1^2 = 1$, and -1^2 also gives +1. So, you have to look for a number that is neither positive nor negative. Can we visualise such a creature? Well, mathematicians around 1450 CE thought that creating such a creature would be useful for mathematics. Three mathematicians – Gauss (Germany), Jean-Robert Argand (France) and Caspar Wessel (Norway) – came up with the notion of the square root of a negative number quite independently. Something was surely cooking in the mathematical consciousness of those times.

This strange number, which could not be placed on the number line, came to be called an 'imaginary number' that was appropriately designated *i*. Visualising this number is no easy task. I did mention my frustrating encounters with negative numbers in grade 7 in school. How could one even think of a number that was less than nothing? For us at that time, nothing was represented by the number zero. So you can imagine how crazy people must have become with *i*. It took much longer to reconcile with *i* than with 'negative numbers'.

How does one visualise this number? Geometrically, we can visualise 'real numbers' like 1, 2, 3, 1.2, 3.4, etc, on a number line, as we saw earlier. These numbers also have a direct bearing on happenings in our world, like we can see 3 dogs or buy 4.5 kg of sugar. Irrational numbers like $\sqrt{2}$, e and π also have a place on the number line, although it is not possible for us to pinpoint their location on the number line, given that they are 'incommensurable' (they cannot be measured exactly, given their endless stream of decimal digits). Still, we can narrow down the location with as much precision as possible.

But *i*? It took some time for mathematicians to realise that i did not have to be anywhere on the real number line. It did not really have a place there. So, a new line had to be made, which was placed at right angles to the real number line as shown in Figure A3.

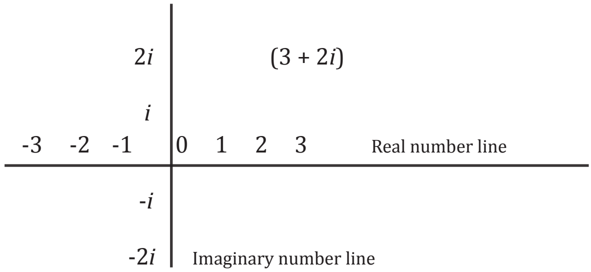


Figure A3

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With this, one can operate with real as well as imaginary numbers in the plane that these two lines create (called the 'Argand plane'). Thus, one can have a number such as (3 + 2*i*) shown above, by moving 3 units on the real number line and 2 units on the imaginary number line. Such a number is called a 'complex number'. Let us leave it at that for the moment. What is interesting is that this complex number, consisting of a real part and an imaginary part, which does not seem to be related to anything in our material world, is used to understand electrical circuits as well as aircraft design among a host of other connections that it has with the real world. That brings me back to what Wigner said about the 'unreasonable effectiveness of mathematics' in explaining the real world. That is the fascinating feature of this subject.

1.4 A note on beauty in mathematics

I'm tempted to talk a little more about the notion of beauty in mathematics. We have Euler's equation to do this in case you feel it got short shrift in the chapter on beauty in mathematics.

The British mathematician Keith Devlin was moved deeply enough to say:

"Like a Shakespearean sonnet that captures the very essence of love, or a painting that brings out the human form that is far more than skin-deep, Euler's equation reaches down into the very depths of existence."

I can well imagine what may have gone on in his mind when he made this statement. To appreciate it, let us visualise the Euler identity (Euler's equation)by writing out these numbers:

$(2.7182818284\ldots)^{(\sqrt{-1})\,x\,(3.14159265\ldots)} + 1 = 0$, or

$(2.7182818284\ldots)^{(\sqrt{-1})\,x\,(3.14159265\ldots)} = -1$

The above equations represent $e^{i\pi} + 1 = 0$

Note that 'raising one number to a certain power' (which is what we do in the above expression) simply means this – if you raise a number 'a' to the power 'n', it means you multiply a by itself n times. So if we say a^3, it would look like this: a x a x a, while a^4 is a x a x a x a, and so on. Note that 'a' is called the 'base' while 'n' is called the exponent. Both 'a' and 'n' need not be whole numbers. Thus, it is fine to have a creature that looks like $2.5^{3.5}$ even if this creature is a bit harder to understand.

There is something very intricate and crazy happening with the Euler identity which we laypersons can only intuitively grasp. You have this irrational e, which when raised to the product of another irrational π and the imaginary *i*, gives -1. This -1 is on the left hand side of the equation so that leaves us with zero on the right hand side. What beats me is how this happens. First of all, you have this e and π whose decimal portions are never-ending or recurring as even the most powerful supercomputers of our age have found. If one can find a way of adding them, it might result in another irrational number. Ditto, if one is subtracted from the other. If they are multiplied by each other, we might end up with yet another irrational number. But if e is raised to the power of the product of *i* and π, then the strangest of things occurs – the infinite strings of digits just disappears, leaving us with -1! And when this is added to +1, as we see in the Euler identity, we are left with nothing. That is when we say that $e^{i\pi} + 1 = 0$.

Benjamin Pierce, the American mathematician, hit the nail on the head when he stated:

> "Gentlemen, that ($e^{i\pi}$+ 1 = 0) is surely true, but it is absolutely paradoxical; we cannot understand it, and we don't know what it means, but we have proved it, and therefore, we know it must be the truth."

That is some consolation, coming from a mathematician! I guess these people are themselves flummoxed many a times. You could spend your entire life unravelling these relationships, like mathematicians did. Touching, indeed, are their heroic stories of struggle for meaning.

Much as Euler's equation intrigues me, what I find interesting is the following: As children, our learning of zero wasn't so much of a problem. We weren't much bothered about it, and just let it be. We encountered problems with it when we learnt place value and also did subtraction with borrowing. But these were procedural issues. That zero could also be equal to $e^{i\pi}$ + 1, containing some of the craziest numbers you can imagine, is nothing short of a revelation.

Well, there are solid explanations and proofs as to why the Euler equation works, like Pierce says. But the explanations actually kill the sense of mystery. When we prove something, its mystery disappears, but only to lead us on to another mystery that will keep the best minds occupied for years to come. Such is the fascinating nature of human discovery. There is really no end to it. If this chain of discovery had ended, our lives would have been so boring. It is the sense of the mysterious that keeps us going on and on. Let that sense prevail. While I'm all for the statement: "Let there be light", I'm also for mystery. In mathematics you have plenty of room for both.

1.5 The infinite series in mathematics

In our minor explorations of e, π and *i* thus far, we have encountered the idea of 'infinite series'. I wish we had learnt about them at school! Anyway, the idea of an infinite series of terms goes all the way back to the fifth century BCE to Zeno the Greek in what is known as 'Zeno's Paradox', a mathematical problem from antiquity. A series as such is a sum of the terms in a mathematical sequence. Series can have a finite or limited number of terms ($s = 1+2+3+4+5$) or an infinite number of terms ($S = 1+2+3+4+5+\ldots$). A series with an infinite number of terms can diverge and be unbounded, as in the second example, or converge, as in the following example:

$S = 1 + ½ + ¼ + 1/8 + 1/16 +\ldots$ Can you guess the number that this series converges to?

The interesting part is that a tool such as the infinite series is very useful in calculating values of numbers, especially irrational numbers up to any level of accuracy we desire. This is perhaps the mundane part. The really interesting part is how these series throw light on the deeper connections between numbers and notions in mathematics, as we have seen in the expression of π.

1.6 No Nobel for mathematics?

This question may already have cropped up in your mind. The story goes that this chap (Alfred Nobel) who invented dynamite and manufactured arms but said he was a pacifist (hence, an irony), created a fund in his will to award Nobel Prizes (starting from 1901) to those: "…who, during the preceding year, shall have conferred the greatest benefit on mankind." Physics, chemistry, physiology or medicine, literature and peace were easily identified as prime areas of human endeavour with the potential to satisfy these criteria. Economics (referred to sometimes as a 'dismal' science) was added to the list much later in 1968. The claim of mathematics for a legitimate place on the list was suspect in this respect, or so goes one facile argument. But the real reason seems to be the intense dislike Nobel had for the Swedish mathematician Mittag-Leffler who represented the University of Stockholm, where everyone thought the prize would be instituted. Nobel gave the institution a miss in his will and there was also a 'Nobel flop' for mathematics.

But mathematics has its own global level prize, equal in prestige to the Nobel Prize. It is called the Fields Medal (established by the Canadian mathematician John Charles Fields, in his will). Started in 1936, Fields Medals are awarded every four years at the International Congress of Mathematicians. Remember Manjul Bhargava?

1.7 Gauss's shortcut

Recall that astonishing example of Johann Friedrich Carl Gauss. The story goes that Gauss's teacher, one Buttner, a 'virile brute', as ET Bell puts it in his *Men of Mathematics*, had barely completed stating the problem when Gauss placed his slate with the written answer on Buttner's table. "Ligget se," Gauss is supposed to have said to Buttner, which basically means in French, "There it lies." This feat of genius completely changed Buttner's attitude towards Gauss. From then on he tried his best to encourage Gauss by buying him the best textbooks available, and also by bringing him in touch with people he knew could engage with Gauss and take him further on his mathematical journey.

Consider the way Gauss had re-arranged the numbers 1–100 as shown below:

1	2	3	4	5	6	…	47	48	49	50
100	99	98	97	96	95	…	54	53	52	51

The product that Gauss obtained was 50 x 101 = 5050. This logic applies to a series of numbers like the above (known as an 'arithmetic series') having any number (say n) of terms. In our example, n = 100. We learn in high school that the sum of an arithmetic series of numbers with n terms is $S_n = n(n+1)/2$. Every high school student knows this commonplace formula. It can be proved by a process called 'mathematical induction'. Interestingly, Gauss's method, too, leads to this formula, since his product 50 x 101 = 50 (100+1), is actually (n/2) (n+1), where n = 100. Now isn't that neat, especially since it came from a 10-year-old?

The point to be noted here is that in an arithmetic series, the difference between the terms is constant. So, you could have a series such as 1 + 4 + 7 +…+ 28 (where the difference between the terms is 3) and still use Gauss's approach to get at the answer. Can you attempt this without using the formula? How would you approach the series 5+6+7+…+37? The issue lies in getting the pair of numbers right. Also note that an arithmetic series can have either an even or odd number of terms.

The sad thing is that we teach the above formula in a dry way and in doing so miss out on Gauss's amazing feat altogether. Many Gauss's lie hidden in our schools, waiting to blossom.

CHAPTER 2

2.1 Challenging the standard history of proof in mathematics

In Chapter 5, we discussed the vast non-Western heritage of mathematics. The Indian contribution (in particular, that of the Kerala school of mathematics) came in for mention. There is now enough evidence to show that mathematics thrived in many cultures since antiquity. As part of the dominant interpretation that places Europe at the centre of global mathematical development, it is interesting to note that a similar 'standard history' of mathematical proof exists. This, too, is Eurocentric.

Euclid was the main hero for us in Chapter 2 and we spent some time in understanding the idea of mathematical proof that is generally attributed to him. The standard description is that mathematical proof emerged from the way the Greeks went about their mathematics in ancient times, especially in the field of geometry. The best examples often cited to make this argument are the works of Euclid, Archimedes and another Greek mathematician called Apollonius. It is also claimed that there was no notion of proof in mathematics before this 'Greek miracle', thereby implying that other non-Western traditions were not really interested in establishing mathematical truth. Some researchers have even talked about the 'intellectual inferiority' of the Orient, compared to the Greeks for whom reasoning and logically-deduced results were often a habit.

The point is that this position has been challenged, and that things are not as black-and-white as they seem. The counter to this interpretation is that many other mathematical traditions are missing from it. Further, the standard history of proof has been mostly discussed in the case of geometry. The idea of establishing truth was important in other cultures as well, as unfolding evidence suggests in areas other than geometry.

It is interesting to note how these stories are created. At a social and political level, history is often used as a tool by one culture to dominate and subjugate other cultures. But that is a different topic altogether.

2.2 Converse of a theorem

One of the things that Channa did with us, which I often found tricky in terms of understanding, was the discussions we had on the idea of the 'converse' of a theorem. This was, as I later ascertained, not a part of our syllabus but brought in by him to illustrate an important aspect of the idea of proof. The idea of the converse of a theorem is illustrated in the following example. Let us take the Pythagoras Theorem. If the converse of this theorem were to be stated, this is how it would look:

"If in a triangle the square of the length of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle."

The exercise of writing the converse statements is itself interesting; one needs to try out many statements to get a grip of the converse. As I have already stated, proving the converse is sometimes more difficult that proving the theorem itself. Secondly, the converse need not be always true, as the following example will clearly show:

Theorem: If two triangles are congruent, then their corresponding angles are equal.

Converse: If the corresponding angles os two triangles are equal, then the triangles are congruent.

Congruent triangles are those that are equal in all respects – sides as well as angles. It is clear that there can be many triangles where each angle of each triangle is equal to the corresponding angle of another triangle. But does this guarantee that their sides are also equal? I suppose this can be figured out quite easily. So, the converse need not be true all the time. An easier way to understand the idea that the converse is always not true is to look at the following two statements:

Statement: If Peter is in Mumbai, then he is in India.

Converse: If Peter is in India, then he is in Mumbai.

That is how the converse works. Or does not work.

CHAPTER 3

3.1 A note on Bhāskara's *Lilavati*

Channa quoted the problem we discussed in class from the Kannada translation of *Lilavati* by Dr KS Nagarajan, published by the Department of Literature and Culture, Government of Mysore, in 1961. I have taken the second *Lilavati* problem (the one that needs to be solved as a quadratic equation) from Joseph's *Crest of the Peacock*. Incidentally, as I discovered, this book is part of Channa's library. So, he must have read it at some time.

CHAPTER 4

4.1 The Seven Bridges of Königsberg

The source of Euler's sketch of the Königsberg bridge problem is Volume 8 of his works, the year being 1736 (published in 1741). I have obtained it online, courtesy 'The Euler Archive' hosted by The Mathematical Association of America.

4.2 The great problems of mathematics

Our maths education in school is bereft of exciting stories, the best parts of mathematics that one should learn in school. I'm referring especially to the history of the subject, its problems and paradoxes and the stories about mathematicians – all of which can make the learning of the subject an incredible adventure. The problem with school mathematics is that it is presented as a set of problems that all have answers. Textbooks present the subject as an open-and-shut case. Thus they slam the door on creativity and imagination.

Mathematical problems, especially the ones that have resisted solution for a long time, sometimes for centuries, like Fermat's Last Theorem, occupy a central place in the development of the field and in the creation of new and more mathematics. As many mathematicians will testify, they are the lifeblood of the subject and a sure sign that it is thriving.

So, is there a list of the all-time great problems of mathematics? Perhaps there is, as the distinguished mathematician David Hilbert wanted us to believe in 1900 when he delivered his now famous address to the International Congress of Mathematicians in Paris. Hilbert outlined 23 problems at the time and said towards the end of his lecture: "Permit me in the following, tentatively as it were, to mention particular definite problems, drawn from various branches of mathematics, from the discussion of which an advancement of science may be expected." Interestingly, while Fermat was mentioned in the lecture, FLT did not figure in the list of Hilbert's 23. There may actually be more than 23 problems because, in some cases, the problems come as a cluster or family of problems needing solutions.

I wish we could discuss Hilbert's problems here. But it would be a difficult task that would take several pages. I'm optimistic that it will be a worthwhile exercise to write about these problems for both children and teachers, just to communicate the excitement of the subject. For now, let me just say that some of them have been solved, while others continue to elude solution.

For a more recent list, the mathematician Ian Stewart identifies 14 all-time great problems which stumped the greatest minds, in *The Great Mathematical Problems* (also recommended as further reading). Some of these still remain unresolved. Stewart includes two problems in his list of 14 that we discussed with Channa – the Four-Colour Problem and Fermat's Last Theorem. In addition, he identifies 12 problems for the future. I suppose a supply of problems (great or ordinary) will continue as long as we keep asking questions.

And finally, great problems in mathematics are also accompanied by money as a reward for their solution. In European mathematics, the culture of awards perhaps began around the sixteenth century. There isn't much written about the possible existence of this practice in other mathematical cultures. Awards for mathematical problems increase their visibility and create more excitement. But I'm not for a moment suggesting that the mathematical community will work harder because there is a good amount of prize money involved. Such as the one million US Dollars per problem without any time limit in the case of the Millennium Prize Problems established by the Clay Mathematics Institute (CMI) in 2000, offered exactly 100 years after Hilbert's famous lecture in Paris. On cracking FLT, Andrew Wiles received the Wolfskehl prize money of US $ 37,000 in 1997. But for mathematicians, the motivations are not monetary, they are mostly intrinsic and there are examples where they have even refused prize money.

In the 'Introduction' of the CMI book on the seven millennium problems identified by the institute, the institute-founders state that CMI seeks to: "further the beauty, power and universality of mathematical thinking." Note the mention of beauty in mathematics, with which we started this book. I must mention the names of these seven problems, though they may not mean much without a discussion: Birch and Swinnerton-Dyer Conjecture; Hodge Conjecture; Navier-Stokes Equations; Poincare Conjecture; P versus NP Problem; Riemann Hypothesis; Quantum Yang-Mills Theory.

Note that two of these millennium problems are directly related to the physical world. One is the set of Navier-Stokes Equations, which govern the flow of fluids such as water and air. The problem here seems to be that we don't know much about the solutions to these equations (there are only conjectures thus far, needing proof). The other problem is the Yang-Mills Theory, also known as the 'mass-gap hypothesis' in a branch of physics called quantum theory, which is a study of the world of the ultra-small (atoms and subatomic particles). The point I wish to make here is that a lot of the developments in mathematics (though not necessarily all of them) are spurred by problems from the physical world. A 'pure' mathematician would have an idiosyncratic view about what I'm saying, because to him or her, the links between mathematics and the physical world are of no consequence.

Finally, it must be added that the term 'conjecture' in mathematics refers to a statement that looks like it is true (you may also be willing to bet your money on it), but that has not been formally proven. Conjectures crop up when one believes that there is a pattern about which one can make a guess but cannot prove as yet.

And talking of patterns, did we not say that mathematics is a study of patterns?

4.3 Is mathematics incomplete?

What a strange question to ask! Especially when we are so used to the 'right answer' all the time, having been brought up on the diet that maths is perfect, logical, blah, blah, blah… And what does this question have to do with what we have been discussing? Well, let's go back to Euclid in Chapter 2. Based on a few axioms (those obvious or self-evident truths), we found that Euclid was able to construct an entire system of geometry that holds good for flat surfaces. We also saw what happens when we do away with one or two axioms – we cannot move an inch forward when it comes to finding results and proving theorems. Thus, all the five axioms are critical.

While it did take another 2,000 years for the momentum to gather around the issue of the role of axioms in mathematics, mathematicians at the turn of the last century got excited with the question: Can we axiomatise all of mathematics like Euclid did in geometry? In other words, mathematicians started looking for a logical system in which all of mathematics could be deduced, like we deduce theorems from Euclid's axioms. All that one needed was a finite set of axioms using the finite rules of logic. We would then have a complete system of mathematics. This, it was thought, would be invaluable for scientific enquiry.

The only two conditions were that such a set of axioms had to be both consistent and complete. A system is consistent when we are not able to prove a statement and its negation. Either the statement or its negation can be proved, not both. For example, if an axiomatic system can prove that quadrilaterals (four-sided closed figures such as squares, rectangles, parallelograms) are made from two triangles and also prove that quadrilaterals are not made from two triangles, we would run into a problem. I hope you are with me on this. The axiom system then contradicts itself and becomes pretty useless. What then distinguishes truth from falsehood in such a system?

Let me come to the second condition – completeness. A system of axioms can be said to be complete if one can prove all true statements and also disprove all falsehoods that arise under the axioms and rules adopted. What if we have a statement that can neither be proved nor disproved? We then have a serious problem and say that the system is incomplete. A good example is the Barber's Paradox, which can neither be proved nor disproved. In fact, it was this paradox that caused much consternation among mathematicians who were searching for a universal set of axioms for all of mathematics.

Enter, Kurt Gödel. Gödel (1906-1978 CE) is considered by many to be one of the most important logicians to have ever lived. He showed remarkably and startlingly, using rigorous mathematical proof, that any axiomatic system of mathematics was incomplete even if it was consistent. This is his famous 'Incompleteness Theorem' (formulated in 1930, when he was 24 years old). It basically means that there will always be statements that can be generated which are unprovable. Now this requires careful reflection, for Gödel is telling us something very significant. In essence, he is saying that every axiomatic system of mathematics will run into problems that it cannot solve. If we take this further, it can only mean that rational thought processes can never reach the ultimate truths, whatever that may be. Well, let me just emphasise one point here – Gödel proved the incompleteness of mathematics using mathematics itself. Gotcha?

Kurt Gödel, the Austrian-American mathematician and philosopher

This gives me goosebumps. For instance, could it mean that the seven CMI problems, including the Riemann hypothesis, can never be proved? More questions can be asked. Flowing from the Incompleteness Theorem, can it then be said that science, which depends so critically on mathematics, will never be able to expose the ultimate secrets of the universe? Remember, this is different from saying, "Knowledge is always advancing…so even if we do not have the answers today, we will find them tomorrow." Mr Gödel places a clear limit on how much we can know, for all time. I wonder what the mystics – East or West

– will say. There can be multiple paths to truth as well. Why should we believe that rational thinking represents the only way?

There is so much to be discussed here, but that will need another book.

CHAPTER 5

5.1 On Vedic mathematics

This topic is a sensitive matter, especially in these charged times when there are vigorous attempts to rediscover our lost heritage and traditions and to also 'Indianise' education. I do not have issues per se here. While I did use some of the tricks of Vedic mathematics, which helped in performing faster cealculations, I was also involved in discussions about the origins of Vedic mathematics, and it does not seem to bear any relationship with the Vedas. The term came into usage following a book misleadingly titled Vedic Mathematics written by one Bharati Krishna Teertha.

By denoting our mathematics as of 'Vedic' origin (for which little or no evidence exists), we may be ignoring the actual and rich heritage of mathematics of the Indian subcontinent, as scholarly works such as Joseph's show. The case for Vedic mathematics therefore is rooted more in the dynamics of politics, religion and power than in the pursuit of historical truth, which should inform what is taught.

5.2 Logarithms

Just in case you thought that logarithms is all about simplifying cumbersome calculations involving huge numbers, let me set the record straight. It is much more than that, though it must be said that the practical issues of computation played a big role in the invention of the idea. In particular, with the advent of logarithms, astronomical calculations and computations in economics, trade and engineering benefitted immensely.

A deeper study of logarithms takes us to the notion of the 'logarithmic function' which in turn finds many applications in real life. So, you can use logarithms to describe the measure of a substance's acidity or alkalinity (measured as its 'pH' level), to describe sound intensity (in decibels) or the measure of earthquake intensity (what the Richter scale measures); they all conform to a logarithmic scale. Then there are wide applications in the study of music, psychology, probability theory, physics, geometry and so on.

The idea of logarithms is a critical one in the study of mathematics. To provide a small example: Gauss used logarithms to arrive at a formula that can tell us how many prime numbers are there for any integer 'x' (that is, the number of prime numbers up to or before 'x' on the number line). The formula looks like $\pi(x) = x/\log x$, where $\pi(x)$ is the number of prime numbers. This formula serves only as an approximation and it must be mentioned that the issue of the distribution of prime numbers is still a famous unsolved problem in mathematics, namely the Riemann hypothesis, which is one of the seven CMI problems with one million dollars at stake. David Hilbert is supposed to have said: "If I were to awaken after having slept for a thousand years, my first question would be: Has the Riemann Hypothesis been proven?"

5.3 Mnemonics for π

We came across one mnemonic for π: "May I have a large container of coffee?" This gives the value of π up to seven decimal places (3.1415926). You can develop your own mnemonics, like I did at The Valley School. They can be a lot of fun. Savour this one: "How I want a drink, alcoholic of course, after the heavy lectures involving quantum mechanics." Here, we can go up to 14 decimal places – 3.14159265358979. Mnemonics like these and several more have appeared in both magazines and journals across the world from time to time. You can churn out your own and see how far you can go. It is a good way to do some creative writing, differently.

CHAPTER 9

9.1 A note on the Steiner-Lehmus Theorem

A few points must be made about the Steiner-Lehmus Theorem that was used by Channa in 1960, during a science exhibition in Sagar to provide a challenge to students and teachers who visited the exhibition. Consider the statement of the theorem: “Every triangle with two angle bisectors of equal lengths is isosceles.” (Recall your understanding of the isosceles triangle, in which any two sides are equal and the angles opposite the equal sides are also equal). The theorem can be represented by the following diagram, where BE and CD are bisectors of angles B and C respectively (bisect = cut into two equal parts).

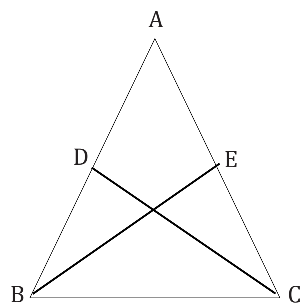

Figure A4

When BE = CD, angle ABE = angle EBC and angle ACD = angle DCB, and the triangle ABC is isosceles. This is what the theorem asserts. Channa posed it as a problem, little knowing that it would take many hours of work to prove. But then, ‘youthful exuberance’ (as he later put it) saw him through and he had a proof worked out before the next day dawned. Surprisingly, the converse of this theorem, which is: “If a triangle is isosceles, its angle bisectors are congruent (equal)” is much easier to prove. Thus, in geometry, sometimes the theorem is more difficult to prove than the converse, and sometimes vice versa. Further, the converse of a theorem need not be true, as we have seen in Chapter 2. Interesting! This is what we will see next.

Whet thy appetite: Channa's 20

OK, the journey is not over as yet. It goes on. Here is Channa's challenge to keep you going on this great mathematical adventure – 20 of his chosen problems covering the areas of geometry, algebra and coordinate geometry, which he solved over the years and whose solutions he regularly submitted to publications such as the *Australian Mathematical Gazette*, the 'Problem of the Week' of Trinity University, *The Mathematical Gazette* of the UK, among others. They are exciting. Some of Channa's solutions were published, some were not.

Whom are these problems intended for? Well, for anyone who has learnt maths in high school. That is what Channa would want – give you a battle, even if you do not fancy yourself to be a maths person. Who knows, you might become an aficionado pretty quickly if you are persistent.

A few things first. The problems are organised in order of difficulty (but the sequence is not sacrosanct), with the easier ones included in the beginning and the more difficult ones coming afterwards. Of course, it does not mean that you work your way through them in this order. You can attack any problem you want, in any order.

One more thing. The mathematics needed to solve these problems is higher than the maths needed to understand the main chapters of this book. But we have still tried to keep it as simple as possible. It is mostly high school stuff (grades 9 and 10), though you will not find these kinds of problems in your textbook. Which is a pity.

One suggestion is that you attempt these problems together with your friends and teachers. Your teachers should be challenged as well. That may make it easier, so you could start solving them pretty quickly. Once that happens, I would suggest that you do what Andrew Wiles did when he went after Fermat's Last Theorem – retire into your private space and mull over the rest, one by one. If that yields you results, there will be this great sense of fulfillment. If that doesn't happen, you could again get back to a larger group and mount a fresh, collective attack.

These are just some suggestions. You are the best judge.

1. Inscribed radius: Let ABCD be a square with an inscribed circle. Let P and Q be points on sides AB and BC respectively, such that PQ is tangent to the circle. If PB is 3 and QB is 4, what is the radius of the circle? *(This problem is from the Australian Mathematical Gazette, September 2015.)*

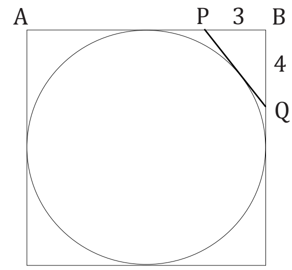

Figure W1

2. It is given that x and y are positive integers, and $(x^2 + y^2)/(x–y)$ is also an integer that divides 35 exactly. Give two solutions which satisfy the above. *(This problem is modelled on a problem from a Bulgarian maths competition in 1995.)*

3. If x, y and z are positive integers, then one solution of
$xy + yz + zx = xyz + 2$ is x=1, y=1, z=1. Can you give another solution? (This problem was picked up from an International Mathematics Olympiad.)

4. In ΔABC, let D be the mid-point of BC. If $\angle$ADB is 45^0 and $\angle$ACD equals 30^0, find $\angle$BAD. Hint: Draw a $\perp$ (perpendicular) from B to AC (please note that the symbol $\angle$ denotes 'angle'). (This problem was picked up from a regional Mathematics Olympiad.)

5. In ΔABC, BP and CQ are drawn to meet the sides AC and AB respectively. $\angle$BCQ = 4x, $\angle$CBP = 3x, $\angle$BQC = 2x and $\angle$BPC = 3x. Prove that AB = AC. (This problem is modelled on a Mathematics Olympiad problem.)

6. Triangle existence: (i) For which integer values of x does there exist a non-degenerate triangle with side lengths of 5, 10, x? (ii) In a triangle, an altitude refers to the perpendicular distance from a vertex to the opposite side. For which integer values of x does there exist a non-degenerate triangle with altitude lengths 5, 10 and x? (This problem is from the *Australian Mathematical Gazette*, July 2014.)

7. ΔABC is an acute-angled triangle. AD, BE and CF are the altitudes drawn to BC, CA and AB respectively. Join E and F. Let EF meet the circumcircle of ΔABC at P. Produce BP and DF to meet at Q. Prove that AP = AQ. Hint: Use the property of a cyclic quadrilateral. (This is an International Mathematics Olympiad problem.)

8. Diagonal difference: In a regular nonagon (a polygon of nine sides), prove that the difference between the longest diagonal and the shortest diagonal is equal to the length of the side of the nonagon. (The problem is from the *Australian Mathematical Gazette*, July 2013.)

9. Floating Fedora: Sammy dives from a bridge into a river and swims upstream for one hour at a constant speed. She then turns around and swims downstream at the same speed. As Sammy passes under the original bridge, a bystander tells her that her hat fell into the water. In order to retrieve the bystander's hat, Sammy continues to swim downstream at the same speed. She finally catches up with the hat when she is exactly one kilometre away from the bridge. Assuming it is constant, what is the speed of the current? (This problem is from the *Australian Mathematical Gazette*, July 2015.)

10. Telescoping product: Let n be an integer greater than 1. Simplify:

$$\left(\frac{(2^3-1)}{(2^3+1)}\right)\left(\frac{(3^3-1)}{(3^3+1)}\right)\left(\frac{(4^3-1)}{(4^3+1)}\right) \text{ and so on, up to } \left(\frac{(n^3-1)}{(n^3+1)}\right)$$

Hint: Use factors of $a^3 - b^3$ and $a^3 + b^3$, where $a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$ and $a^3 + b^3 = (a+b)\,(a^2 - ab + b^2)$. (This problem is from the *Australian Mathematical Gazette*, May 2013.)

11. Prickly pair: I'm thinking of a pair of numbers. To help you work out what they are, I will give you some clues. Their difference is a prime, their product is a perfect square and the last digit of their sum is 3. What can they possibly be? (This problem is from the *Australian Mathematical Gazette*, September 2013.)

12. Eight coins weighing 1, 2, 3, 4, 5, 6, 7 and 8 grams are given, but which one weighs how much is unknown. A man claims he knows exactly which coins are which, and as an offer of proof, he claims he can perform a

single weighing on a balance scale so as to unequivocally demonstrate the weight of at least one of the coins. Is this possible or is he exaggerating? (This problem is from 'Problem of the Week', Trinity University.)

13. If $x^2 + x + 1 = 0$, compute the numerical value of:

$(x + 1/x)^2 + (x^2 + 1/x^2)^2 + (x^3 + 1/x^3)^2 + ... + (x^{27} + 1/x^{27})^2$

Hint: $x^2 + x + 1 = 0$ implies that $x+1/x = -1$. Using this, find x^2+1/x^2, etc. Don't use the 'brute' method to compute each of $x^3 + 1/x^3$, $x^4 + 1/x^4$ all the way up to $x^{27} + 1/x^{27}$. Study the pattern of values of a few initial expressions and proceed. (This problem is from 'Problem of the Week', Trinity University.)

14. 10 (not necessarily distinct) integers have this property that if all but one of them are added, then the possible results (depending on which one is omitted) are: 82, 83, 84, 85, 87, 89, 90, 91, 92. What are the 10 integers? (This problem is from 'Problem of the Week', Trinity University.)

15. Tangent intersections: Let C_1 and C_2 be two non-overlapping circles with centres O_1 and O_2 respectively. From O_1 draw the two tangents of C_2 and let them intersect C_1 at points A and B. Similarly, from O_2 draw the two tangents to C_1 and let them intersect C_2 at C and D. Prove that AB = CD. (This problem is from the *Australian Mathematical Gazette*, May 2013.)

16. Drawing parallels: Two | | (parallel) lines are drawn on a sheet of paper. There is also a marked point which does not lie on either of these lines. Here is your challenge: using an unmarked straight edge (and no compass), construct a new line through the marked point, that is also | | to the existing lines. (This problem is from the *Australian Mathematical Gazette*, July 2014.)

17. ABC is an equilateral triangle. CE is drawn $\perp$ to AC at C. A straight line is drawn through A meeting CE at E and BC produced at D such that ED = AB. Prove that, if AB = 1 then $(AE)^3 = 2$. (This problem is taken from *The Mathematical Gazette*, UK, 1993.)

18. Let ABCD be any plane quadrilateral with A (0, 0), B (p, q), C (b, c) and D (a, 0). Let a parallelogram be created by constructing through the ends of each diagonal of ABCD, lines parallel to the other diagonal. Show that each diagonal of this parallelogram passes through the intersection point of a pair

of opposite sides of ABCD. Hint: Use the methods of coordinate geometry for collinear points. (This problem is modified from the *American Mathematical Monthly*, January 2016.)

19. ΔABC is isosceles with AB = AC and centroid G. Let D be the point (other than G) on the circumcircle of ΔBGC such that BD = CD. Show that the centroid of ΔBDC lies on the circumcircle of ΔABC. (This problem is taken from *The Mathematical Gazette*, UK, 2015.)

20. Let PQR be a triangle whose circumcircle C and incircle D have radii R and r respectively. If d is the distance between the centres of C and D, prove that: $(R-r) < (r+d) \leq (R-r)\sqrt{2}$. (This problem is taken from *The Mathematical Gazette*, UK, 1996.)

Further reading... further reading... further reading

This bibliography is not meant to be thorough. But it has some general readings that I believe will take you further. Not all the books I have consulted are not included in this list.

ET Bell's *Men of Mathematics* (Simon and Schuster, 2008) provides an enjoyable introduction to those interested in knowing the mathematics, and discovering the fascinating, often quirky lives of some 40 mathematicians from Zeno (fifth century BCE) to Georg Cantor (twentieth century CE). This mammoth volume features only European and Western mathematicians though. Also, Bell has been criticised for distorting facts and presenting an idealised picture of the lives of mathematicians. In addition, he provides no space for the achievements of women mathematicians. Still, *Men of Mathematics* is an entertaining read.

For those who want to know the entire story of Fermat's Last Theorem (FLT) – from its beginnings to the day that Andrew Wiles cracked the riddle – I highly recommend the popular accounts by Simon Singh and Amir Aczel. Their books have the same title: *Fermat's Last Theorem* (Fourth Estate, 1997 and Delta, 1996, respectively). Both books combine great storytelling with the history of this discovery. Also, do not miss watching the film on FLT by Simon Singh (produced by the BBC). It is one of the few films on mathematics that holds your attention till the end. You will see what can make a great mathematician burst into tears.

For history buffs, Howard Eves' *An Introduction to the History of Mathematics* (Holt, Rinehart and Winston, 1969) is a good starting point. But it requires knowledge of high school maths and basic calculus. While there are chapters on the mathematics of cultures other than that of the West, the skew is clearly in favour of the latter. The historical accounts, though, are engaging.

George Gheverghese Joseph's pioneering work, *The Crest of the Peacock* (Affiliated East-West press, 1995), moves you to acknowledge the vast heritage of non-European and non-Western mathematics. His lecture at The Valley School in 1994 was an eye-opener for me. The book is a delightful read, with nicely illustrated chapters on African, Babylonian, Arab, Indian and Chinese contributions. Even if you are not confident of the maths (basic high school maths knowledge), the historical narrative is gripping.

If you want to sink your teeth into some great mathematical problems from antiquity to this day, Ian Stewart's *The Great Mathematical Problems* is the best bet. Problems that have resisted easy solutions, sometimes over centuries, have spurred the development of the subject in many different directions. Stewart identifies 14 mathematical problems that have stumped mathematicians, sometimes for hundreds of years. The book includes detailed discussions on two of the four problems that Channa discussed with us – the Four-Colour Problem and FLT. He also identifies 12 problems that need to be tackled in the future. Knowledge of high school maths will help you wade through this book.

One of the best popular accounts of the idea of infinity is Rudy Rucker's *Infinity and the Mind* (Penguin, 1997). Rucker takes you on a grand tour of what is easily the most fascinating idea in mathematical thought. With Channa, we did discuss infinity when dealing with sets. I remember arguing with him that the number of sand particles on a beach is infinite, because we can, in principle ,keep cutting these particles without end. His argument was that we do not do this in practice, therefore the number of particles remains finite. Anyway, do read Rucker. *Infinity and the Mind* boggles the mind indeed.

The Math Gene (Phoenix, 2000) by British mathematician Keith Devlin is the odd one out in this short list of books on mathematics, as it focuses more on the issues of teaching and learning mathematics than on the features of the subject itself. I'm including it here because the book asserts that: "everyone has it (the math gene), but most people don't use it." Devlin goes on to show that mathematical ability, much like the ability for language, lies within each and every one of us and that this ability can be honed. *The Math Gene* is a superb read for all of us who believe that everyone can partake in the adventure of mathematics, as Channa so strongly believed. In fact, this is the very reason why I wrote this book.

If you are just setting out and want to whet your appetite for mathematics, you should make *What is Mathematics? An Elementary Approach to Ideas and Methods* (Oxford University Press, Second edition, 1996) by Richard Courant and Herbert Robbins your constant companion. The book makes clear, in as lucid a manner as possible, all the fundamental ideas and methods of mathematics. The authors develop their ideas gradually and in a manner that is not intimidating. Then there is quite a bit of history of the mathematical ideas that they discuss as well. Ultimately a serious engagement with *What is Mathematics?* will take the reader to a place from which it will be possible to set off on a great intellectual adventure.

Acknowledgements

There are many who have helped me, in one way or another, in writing this book and to whom thanks are due.

It all goes back to a conversation in the summer of 2012. The late Vinod Raina, key architect of India’s landmark Right to Education Act, was in Raipur. Both of us were to present our views on the Act in the Chhattisgarh Legislative Assembly. I had e-mailed the article I had written on Channakeshava which was to appear in *Voices*, the NCTE’s online journal, to Vinod a few weeks before the event. “It took me down memory lane,” said Vinod, who was a theoretical physicist before he decided to dedicate his life to the cause of education. I could see why he liked the article. “But you can make it even more interesting,” he added. “Make it more informal, conversational, and also illustrate it. I will send you a book that will give you some good ideas.”

That book never reached me because we lost Vinod to cancer the following year. But my conversations with him remained with me and provided the spark that resulted in the book you are reading. They provided me with the self-belief that the article could be expanded into a full-length book for parents, teachers, children and anyone interested in mathematics and education. Thank you, Vinod, for bringing me onto a path that has enriched me immeasurably.

Channa is the central character of the story, the raison d’être for this book. My guess is that he was amused when he first heard about it, wondering what the fuss was all about. But he enthusiastically supported it. We exchanged many e-mails, had several chats both real and online during the writing of the book. These exchanges helped me in clarifying my never-ending questions and doubts. Patiently, as is his wont, Channa stayed the course and watched the unfolding of the manuscript. As always, he gently nudged, shared, explained, clarified, and also threw challenges from time to time.

Thank you, Mr Channakeshava, for being a great inspiration. I hope you have fun reading the book. Please get ready for the children's version which is due next.

Channa's family always welcomed me and was happy to answer my stream of questions, both trivial and serious. A big 'Thank you!' to his wife Sharadamma, his grandchildren Suveer and Aishwarya, his daughters Ranjini and Rohini, and his son-in-law Ramachandra who shared many different facets of the man that I didn't know of earlier.

I wish to thank my fellow Baldwinians, senior and junior, for promptly and kindly sharing their time as well as their memories of Channa by meeting with me, through phone calls and transcontinental correspondence: Vijay Kumar Bommu, Sridhar Chintapatla, Kishen Bhagavan, Mahesh Dattani, Rohan Joshi, Suresh Menon, Goverdhan Jayaram, Ravi Ramu, Navam Pakianathan, Joydeep Sengupta, Abraham Cherian, Biju Sam Jacob, Giri Balasubramaniam, Unnikrishnan Menon, Tejas Shah and Gurumurthy Arumugam. Their recollections of Channa, touching and reflective, helped me validate my own observations about our wonderful teacher. I also appreciate the generosity of the Baldwin Alumni Association, who allowed me to present the first draft of the book to Channa at the old students' reunion that was held at The Windsor Manor in Bengaluru in December 2014.

A special thanks to Sanaulla, our Hindi teacher, Channa's best friend and colleague in Baldwins, for his reminiscences about Channa and about our alma mater.

I wish to acknowledge Swapnil Gaikwad for taking the time to create the two beautiful illustrations depicting the 'π tail' and the number line.

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This book has survived one house-shifting and several hits and misses at work in the course of its writing. I would like to thank my wife Savitha for ensuring that in the face of all these changes I did not lose sight of what was needed to be done, and I would like to thank our children, Neha and Nikhil, for not giving up on the hope that this book will one day be a part of their little library. I hope they read it too.

Finally, I remain indebted to my anna, all of 92, for insisting many years ago that for me, it had to be Baldwins. He still dreams for me.

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A message from the publications team of Eklavya

We dedicate this book to Rex D'Rozario.

Having worked in the Hoshangabad Science Teaching Programme first with Kishore Bharati and then with Eklavya, Rex was subsequently instrumental in giving shape to Eklavya's publication efforts and science popularising initiatives. Between editing mainstream science and news magazines, he always found time to contribute to our work, both officially and as a friendly guide. Rex passed away on January 19, 2017. This was one of the last books he edited for us.

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