Qutub Minar in Bhopal?

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It all started during a discussion over tea. After all, most great discussions take place only over tea and coffee. A popular furniture shop had come out with an ad saying that they had made the tallest chair ever, and that it was on display in their showroom in Bhopal. In the ad they had someone asking for directions to Bhopal and he was told – "Can you see that tall chair? Just set your sights on it and keep going in that direction." Even though the chair (according to the manufacturers) was taller than a two-storey building, would anyone actually be able to see it before entering Bhopal? Would it be possible for us to see it from Indore (where we were having this discussion over chai)? Even if we assumed that there was nothing to obstruct our view between Indore and Bhopal, and our eye-sight was good enough to see something at a distance of approximately 180 km (the distance between Bhopal and Indore), would we be able to see this chair from Indore? It got us thinking.

We realized that for any object to be visible over such a long distance, we had to take one important factor into consideration: the curvature of the Earth – the fact that we are not on a flat surface. To be able to see anything over a very long distance, we have to look around a curve as the earth's surface is curved. Or, the object had to be tall enough for it to be visible above the horizon. So, our question then became - given perfect eye-sight and obstruction-free view between Indore and Bhopal, how tall did a chair (or a building, any object) in Bhopal have to be to be visible from Indore?

Our explanation and solution follows. But why don't you first try and think of a solution to this problem?

This is how we went about answering our question. For the sake of simplicity, we assumed that the earth is a perfect sphere. Now, if Indore and Bhopal are two points I and B respectively, on this sphere which has a centre O, we can always cut this sphere into two halves using a plane that passes through these two points as well as the point O, the centre of the earth (Fig 1a). Recall from your early geometry classes that any three points in a three-dimensional space can form a plane. Then if we take just one half of this sphere, one of its projections will look like a circle as shown in Fig 1b.

Figs 1a, 1b

In case you have any difficulty in understanding this geometry, take a lemon and a knife. Mark any two points on this lemon and cut it into two equal parts by keeping your knife in such a way that it passes through both the points and the centre.

Once we had a sphere as in Fig. 1b, our next task was to make sense of what was known and what was to be calculated. If Indore and Bhopal are represented as points I and B on the circumference of the circle formed by the flat surface of the hemisphere, then the length of the arc joining these two points will be about 180 km (Indore-Bhopal distance). Now consider the tallest building in Bhopal is 'h' meters high. This has been shown as AB in Fig 2.

Fig 2

Since the buildings are perpendicular to the ground, the line segment AB is like a normal drawn on the circle. If you extend the line segment AB inside, it will pass through the centre of the sphere.

To reiterate, for a person from Indore to see the Sun or the Moon is very easy because it is very very high in the sky. But to see something on the ground, we need to consider the spherical shape of the earth. There is a limit to the line of sight beyond which we cannot see as the surface of the Earth curves downwards. So, if we have to be able to see a building in Bhopal, this line of sight should pass through the top of the building i.e. point A – (the line IA in Fig. 3). The line IA represents the line of sight of a person standing at Indore. As you look farther down towards the horizon, you see that the lines come closer and closer to the tangent drawn at point I (T1-T2). The line of sight can not go down any farther. Don't forget that the person's height is negligible in comparison to the distances we are talking about.

Fig 3

Now suppose that the arc IB subtends an angle θ at the centre of the earth. The distances OI and OB are equal to the radius of the earth which was taken to be roughly 6400 km. To work out this angle, we thought that since the whole circumference of the circle subtends 360 degree at the center, then how much angle will be subtended by a part of the circumference – arc IB i.e. 180 km.

Fig 4

Circumference of the earth = 2* pi* Radius of the earth
= 2*3.14*6400 kilometers
= 40212.39 kilometers
= ~ 40212 kilometers

Since 40212 km length subtends the angle 360° at the centre,

1 km arc length will subtend the angle = (360 / 40212)°

=> 180 km arc length will subtend the angle = (360 / 40212) x 180°

=> θ = 1.61°

You will notice that the length proportions and the angle subtended at the centre by arc IB look very different in Fig. 4 from the values we worked out through calculations. The angle subtended at the centre by the arc IB probably looks larger than it should. This is deliberate – we have done this so that the illustration is clearer and the explanation that follows is more easily understood.

Moving on to using basic trigonometry, consider the right angle triangle AIO (since the line IA is the tangent at the point I, and the line IO will be perpendicular to the tangent; OA is the line that is formed when OB is extended to point A).

cos θ = Base / Hypotenuse (by definition)

=> cos θ = OI / OA …...................... (1)

Since, OI is the radius of the earth, OI = ~ 6400 km = 64,00,000 meters

Now, OA = OB + BA

OB is also the radius of the earth, so OB = ~ 6400 km = 64,00,000 meters

BA is the height of the building. So, BA = h meters

Putting all the values in the equation 1,

cos θ = 64,00,000 / (64,00,000 + h)

Since θ = 1.61°, cos θ = 0.9996. (You will have to read the standard 'cos' table for the same.)

=> $0.9996 = OI / (OB + BA)$
=> $0.9996 = 64,00,000 / (64,00,000 + h)$
=> $0.9996 \times (64,00,000 + h) = 64,00,000$
=> $(64,00,000 + h) = 64,00,000 / 0.9996$
=> $64,00,000 + h = 64,02,561.02$
=> $h = 64,02,561.02 – 64,00,000$
=> $h = 2,561.02$ meters
=> $h = \sim 2.5$ kilometers

That is, a building in Bhopal would have to be about 2.5 km high in order for it to be seen from Indore. Doesn't seem like much, does it? But think of it this way, in most of our buildings, the average floor is just 3 m, so a three-storey building is about 10 m. It would take a hundred of such buildings, one on top of the other to get just one kilometre high. So we hope now you are impressed with how high a building in Bhopal would have to be for it to be seen from Indore!

Now, you can try and crack the following puzzle. At what distance would you actually see that tall chair?