Appendix 8

Report of the fractions workshop held at Hoshangabad

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A large majority of the teachers were primary school teachers- teaching class 1 &2 and sometimes up to 5. A few were middle school teachers. A couple of B.Sc students also joined the workshop. Most of the teachers belonged to lower class, rural background and were most likely first generation learners.

Sabyasachi Mitra, now a freelancer but was with Jodogyan till recently, and Arindham, Lecturer, Dept of Mathematics, Patna University came as resource persons for the workshop. Mukesh Malviya, a teacher ,who had been associated with eklavya for long, was involved with the Prashika project and is currently part of the SCERT team, was also present. Rashmi attended the whole workshop and made detailed documentation- partly in Hindi and partly in English. We need to really sit on it, translate the whole thing in English and have a full report.

The object of the workshop was to get the teachers to understand what a fraction is and how to interpret word problems involving fractions and how to solve them. We tried to redefine the four operations in such a way that it makes sense in the context of fractions as well. I used a set of concept oriented word problems I had written up earlier. I have never used these problems while teaching children but I thought that they were ok for teachers (adults) at least. Rashmi felt that the word problems helped build a corpus of experiences. Examples were multifarious and hence one felt recharged each time one had to solve a problem. On the other hand, each situation was complex; every 15 minutes a new situation had to be dealt with and so it was also overwhelming.

We began the workshop by asking for some real-life situations where they encounter numbers other than natural numbers. The next exercise was to represent some word problems in terms of pictures. While discussing this, it transpired that for most people the fraction m/n represents m things out of n things and hence it has to be always a proper fraction. We had to repeat the definition of the numerator and denominator several times during the workshop: the denominator 'n' tells us that the whole is to be divided into 'n' equal parts and the numerator 'm' tells us that we require 'm' pieces of size 1/n. We also had to again and again reiterate the fact that we fix a whole once and for all in a given context. Mukesh was trying to argue that in real life we never deal with something like 11/3- it is always 3 and 2/3- and so there is no need to introduce fractions like 11/3. His argument was that treating 1/3 and 11/3 simultaneously as fractions is not natural; it is not how people think about fractions. It took some effort to convince him that we arrive at such fractions while adding and multiplying fractions and hence it is necessary to have a general enough definition.

We considered three different ways in which the fraction 2/5 can be interpreted- part of a whole, equal share, operator on a collection. Though we repeated these three different situations many times, I am not sure that we managed to convey that irrespective of which interpretation is meaningful in a context, the numerical facts remain the same-- 2/3 > ¼ irrespective of which interpretation we opt for.

One session was spent in figuring out if all ¼ parts of a rectangle will be equal- this was done by paper folding, then cutting and pasting to check if the area remains the same. Participants enjoyed this exercise very much. I thought we used the fraction kit extensively (though Rashmi felt we did n't use it much to solve multiplication and division problems) . Part of the exercise was to make a fraction kit.

We used the idea of defining unit fractions, then composite fractions as sum of unit fractions-- we justified the symbol + by saying that once the size of the piece is fixed, what we are doing is only counting and so 1/3+ 1/3 is two pieces of size 1/3 and hence is equal to 2/3 because by definition, 2/3 is two pieces of size 1/3. Comparison was done step by step-- same denominator, same numerator, comparison with ½ and so on and finally by either equalizing the denominator or numerator. We used the fraction chart to compare fractions, to identify some equivalent fractions.

It took us four days to cover all these and arrive at addition and subtraction of fractions. We spent a little time on why we used LCM while adding fractions and what is meant by LCM. Participants started finding multiples, common multiples and then LCM rather than using the algorithm.

There was very little time left for doing multiplication and division. We explained the 'x' symbol in the context of fraction as a generalization consistent with how it has been used in the context of natural numbers, used pictures to show 1/3 of 2/5 is the same as 2/5 of 1/3 and so on. Division was quite hard, and I feel, we didn't really manage to make them see why 1/2÷ 2/3 means how many 2/3 make ½. Number line and decimals were just touched upon.

We began the workshop with a test-- we used a test paper designed by Ravi Subramaniam for the summer camp at HBCSE leaving out some parts and at the end of the workshop repeated the same test. Administering this paper was probably questionable, given that it basically focused on the multiplicative relationship between two numbers and so addition and subtraction didn't figure in it, but I didn't have time to design a new one and get it translated.

If we go by the performance in the pretest and posttest one can see some progress – people started using pictures, were careful to take wholes of the same size when required, were able to represent improper fractions correctly in pictorial form and associate the right fraction with the shaded region in a figure.

We had three sessions per day-- 7.30 to 9.30, 10 to 1 and 3.30 to 6. This must have tired out the participants but they never said so even when we asked them repeatedly. They had just one week's time and it seemed they wanted to learn as much as possible. They worked very hard, participated well. By the end of the workshop, everyone knew they had to be careful about what the whole is, were able to show some improper and proper fractions using the fraction kit and so on. But I suppose we cannot claim that they moved beyond the fraction kit/ pictures.

Certainly it was too short a time to learn so much. It would have been better if over a long period we met for just a couple of hours per day. Getting this one week itself was difficult- usually we get only 2 days. So I don't know how else a workshop on fractions for teachers can be organized. We hope to follow up so that some of what they learnt gets consolidated and gets used in the classrooms.

Report of the Bhopal workshop on Mathematics, June 2007

The workshop on mathematics for 2007 was held between 20th and 25th June. The workshop was in two parts. Part one consisted of addressing issues in teaching place value and number operations in primary school and was conducted by Sabyasachi Mitra and me. The objective of this was to enable the teacher with activity-based teaching. The need for incorporating cognitively rich activities in the classroom teaching has become particularly important for schools that follow the NCERT textbook for class III and IV. Part two of the workshop focused on teaching fractions. K.Subramanian and the team (Shweta and Smitha) from HBCSE conducted this part.

The workshops started at 10.30 in the morning and went on till 4.30 in the evening with a half-hour lunch break. Primary and middle school teachers from some of the schools in Bhopal and members from Muskan and Samavesh attended the workshop. The total number of participants was about 30. Veena Bhatia and Shubra Guha from the Eklavya WATIS team attended the workshop. Arindam Bose participated in the workshop from the 22nd to 25th. Shika from Wipro documented the whole workshop. Prakash Iyer from Wipro also attended part of the workshop.

On the first day of the workshop we began with a small test on estimation and division problems. Teachers had five minutes in which they were to answer the questions. The papers were checked during the lunch break and the teachers' responses were discussed in the evening. The workshop began with discussing place value based number system and its precursors. We stressed the three different aspects of learning numbers- (i) the act of observing difference in quantity and the act of counting, (ii) learning and using number words for counting (iii) representing number words in terms of numerical symbols. We traced a very short history of writing numbers pointing to non-place value based systems such as Roman number system and the early Indian number systems that were in use before the we started using the Arabic Numerals, and the place value based number systems that were used by Mayans and by Chinese. We discussed how operations of addition and subtraction were done in these different systems before the right left algorithm that we teach today came into being. We also discussed the common mistakes children made in writing numbers and doing number operations on them and why they committed these mistakes. In order that the teachers could appreciate the difficulty children faced in place value based number system, we gave them the 20 symbols used by the Mayans and asked them to write various numbers in the new place valued based system. First exercise was to write the numeral for 'twenty' which calls for introducing a new place. This was followed by questions such as write that largest two digit number or decode a given number. The teachers participated actively in this exercise and saw the difficulties they faced in the new system. Some time was spent in discussing the role played by language in causing confusion in writing numeral and also in the different bases that are in use even today-- such as base twelve and sixty in time measurement. We discussed if we should use 'kadi-bundle' and groupng methods to enable children write

numbers. The teachers were given a copy of Kamii's paper with a brief description of what is discussed in the paper and asked to read the paper and come back with comments the next day.
Sabyasachi discussed the responses to the test. He explained what role estimation played in doing arithmetic and the pitfalls in following algorithms mechanically in division problems. Teachers shared their experiences in all these.

On the second day we started with discussing Kamii's paper. The main points of the paper were highlighted and opened for discussion. There was a long discussion on what constituted activity-based teaching and if we should adopt the activity based approach or an abstract number approach. One of the teachers argued strongly against activity based approach saying that numbers are abstract and using pictures or objects limited the abstract nature of numbers. Other participants commented on their experience of why using activities and objects were necessary for children to understand what they were doing. Shika mentioned the different stages in a child's understanding and how activities were important for children to make sense of what the numbers represented. The rest of the day was spent in doing and developing some activities to enable children to read and write two-digit numbers-- the game of tambola was one of them-- and activities to know the ordering of numbers. The use of counting beads as a precursor to number line was discussed.

On the third day substantial amount of time was spent on understanding how to use the *ganit mala* for counting, representing and recognizing order among numbers. Number board activities and activities to find out if a number is odd or even were discussed. Some time was spent on discussing how the use of word problems as the starting point and encouraging children to find their own ways of carrying out the operation and recording the answer enabled them to become comfortable with numbers.

The next three days of the workshop were spent on leaning fractions. K.Subramnaian also gave a test to asses the teachers' understanding of fractions, then began with why we needed fractions and showed how in the context of measurement counting numbers did not suffice. He discussed the need for fixing a whole and then coming up with other smaller units to measure with. So the unit fractions were introduced and all other fractions were explained as composite fractions-- made of unit fractions. Comparison of fractions and equivalence of fractions were discussed. He also showed how a fraction is the result of a division problem in the sharing context and related the symbol 2÷3 with the fraction 2/3. Teachers were given worksheets to find fractions; they were given different unit fractions of the whole, and were asked to choose an appropriate unit to measure different given lengths and represent them as fractions. On the second day of the workshop teachers were asked to make a fraction kit that they could use in the classroom. This called for some discussion and rich benefit for teachers as making 1/7 was a hard part that required cleaver ideas. Comparing fractions by using the size of the piece, when the number pieces were same, by using number of pieces when the size was same, by comparing the remainder with the whole, by comparing with half were all discussed, and finally we talked about how to use the sharing context for comparing . On the last day teachers listed the important themes in teaching fractions- this served more as a revision of what had been done in the previous two days.

The second half of the last day was spent in tying up different stands and in recalling some of the issues in teaching place value and number operations. We also listed some of the material, which are easily available, for use in the classroom.

In my view:

During the first three days of the workshop much of the time was spent in discussing issues though we also did a few activities. . Sometimes discussions went into the nature of mathematics, inductive and deductive methods of validation, rigour in writing definitions and so on. While all of these are important, it would have been better to spend more time on coming up with activities. Teachers showed a lot of interest in the different books – HBCSE, Kushikushi and NCERT-- that we made available for them, but sadly they did not get much time for going through these books and selecting activities from them. The need to differentiate between activities that are rich in cognitive content and activities that are of limited value was only touched upon very briefly. This was clearly because many a time discussions went off on tangents. We were not able to stick to the plan.

Fractions part of the workshop went very well. My main contention is that K.S took a more logical, rather than activity-based, approach. But having sat through the children's camp at HBCSE, it seemed to me that even children were able to adopt the activity-based approach. A thought still nags at me whether the participants were learning by just following the given line of thought or by critically arriving at an understanding themselves.